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OFFSET FRACTIONAL FOURIER TRANSFORMAND ITS OPERATORS

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ABSTRACT

The Fractional Fourier Transform (FRFT) belongs to the class of time-frequency representation that has been extensively used by signal processing community. The most possible application of the FRFT are optical signal processing, quantum mechanics, optimal filtering. The Offset Fractional Fourier transform is the space shifted frequency modulated version of original one. Aim of this paper is to present two-dimensional Offset Fractional Fourier transform in distributional generalize sense. Also operators on the testing function space E and its dual space E^* *are obtained.*

Keywords: Fourier transform; Fractional Fourier Transform; Two-dimensional Offset Fractional Fourier transform; Generalized Function; Testing function Space.

I. INTRODUCTION

In the present era, different theories of integral transforms have been proposed by the help of integrals with different kernels, range of integrals, chosen suitably. The most useful significance of integral transforms lies in the fact that they transform a class of differential equation into class of algebraic equations, so that solutions of these differential equations can be obtained easily. The distributions are also known as generalized functions which generalized the idea of classical functions and allow us to extend the concept of derivative not only to all continuous functions but also to the discontinuous functions in the classical sense [1].

The Fourier Transform is one of the most important mathematical tool in physical optics, optical information processing, linear system theory, and some different areas of science and technology. The Fractional Fourier is also a linear transform generalized from the conventional Fourier transform. Many interesting properties of Fractional Fourier transform are well known in solving problems in quantum physics, optics and non-stationary signal processing , specially for chirp signal processing and are applied into information encryption and image processing [3] [4] [8]. Offset Fractional Fourier transform is useful in optics. They are especially useful for analyzing optical system with prisms or shifted lenses [7].

In our previous work we have defined the two dimensional Offset Fractional Fourier transform, testing function space E and as follows

1.1 Two-Dimensional Offset Fractional Fourier Transform:

Two-Dimensional Offset Fractional Fourier Transform $[F_\alpha^{\tau,\eta,\xi,\gamma} f(t,x)](s,u)$ of function f (t, x) through an angle α is defined as

$$[F_\alpha^{\tau,\eta,\xi,\gamma} f(t,x)](s,u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x) k_\alpha(t,s-\eta,x,u-\gamma)\,dtdx\,,$$

where$K_\alpha(t,s-\eta,x,u-\gamma) = \sqrt{\frac{1-i\cot\alpha}{2\pi}}\, e^{i(s\tau+u\xi)}\, e^{\frac{i}{2\sin\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]}$

1.2 Testing function space E:

An infinitely deferential complex valued smooth function on $\phi(R^n)$ belongs to $E(R^n)$, if for each contact $\subset S_{a,b}$, where

$S_{a,b} = \{t; x: tx \in R^n\}$; $|t| \le a, |x| \le b, a < 0, b < 0\}$, $I \in R^n$

$$\gamma_{I,l,q}(\phi) = \sup_{t,x,\in I} \left| D_{t,x}^{l,q} \phi(t,x) \right|$$

$< \infty$, $\quad l,q = 0,1,2 \ldots \ldots \ldots$

Thus $E(R^n)$ will denote the space of all $\phi \in E(R^n)$ with support contained in $S_{a,b}$.

Note that the space E is complete and therefore a Frechet space. Moreover, we say that f is Offset Fractional Fourier transformable if it is a member of E^*, the dual space of E .

In the present work, Two-Dimensional Offset Fractional Fourier transform is extended in distributional generalized sense. Operators on the Testing function space E and dual space E^* as Shifting operator, Differential operator, Adjoint operator, Adjoint Differential operator,Adjoint Shifting operator.

II. DISTRIBUTIONAL TWO-DIMENSIONAL OFFSET FRACTIONAL FOURIER TRANSFORM

The Two-Dimensional Offset Fractional Fourier Transform $[F_\alpha^{\tau,\eta,\xi,\gamma} f(t,x)](s,u)$of generalization function $f(t,x)$ through an angle α is defined as, $[F_\alpha^{\tau,\eta,\xi,\gamma} f(t,x)](s,u) = \langle f(t,x)\ K_\alpha(t,s-\eta,x,u-\gamma)\rangle$

where$K_\alpha(t,s-\eta,x,u-\gamma) = C_{1\alpha}\, e^{i(s\tau+u\xi)}\, e^{C_{2\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]}$,

$C_{1\alpha} = \sqrt{\frac{1-i\cot\alpha}{2\pi}}$ and $C_{2\alpha} = \frac{1}{2\sin\alpha}$

III. SHIFTING OPERATOR

3.1. If$\phi(t,x) \in E$ and m, n are real numbers then $\phi(t+m, x+n) \in E$, $t+m > 0$, $x+n > 0$

where $\phi(t,x) = K_\alpha(t,s-\eta,x,u-\gamma)$

Proof: Consider $\gamma_{E,l,q}\phi(t+m,x+n) = \sup_{t,x,\in I}\left|D_{t,x}^{l,q}\phi_\alpha(t+m,x+n)\right|$

$$= \sup_{t,x,\in I}\left|D_{t,x}^{l,q}K_\alpha(t+m,s-\eta,x+n,u-\gamma)\right|$$

$$= \sup_{t,x,\in I}\left|D_{t,x}^{l,q}K_\alpha(t',s-\eta,x',u-\gamma)\right|$$

$$< \infty$$

Therefore for t', x', s, u $\in$ and any fixed l and q, $0 < \infty \le$

Thus $\phi(t+m,x+n) \in$, $t+m > 0,\ x+n >$

3.2. The Translation (Shifting) Operator

$T: \phi(t,x) \to \phi(t+m,x+n)$ is topological automarphism on E for fix $t+m > 0,\ x+n > 0$.

3.3. If $\phi(t,x) \in E$ and $m > 0,\ n > 0$ then $\phi(mt,nx) \in E$

Proof: The proof is simple and hence omitted.

IV. PROPOSION

If $\phi(t,x) \in$ and $m > 0,\ n >$ the scaling operator $R: E -$ define by $R_\phi =$, where $\psi(t,x) = \phi(mt, r$ is a topological automatism. Combining result (3.2) and (3.3), immediately yields next proposion .

V. PROPOSION

If m, n, $r, \wedge \in$ such that $t+m > 0,\ x+n >$ and $rt >$, $\wedge x >$ then the shifting scaling operator $S: E -$ defined by $S(\phi) =$ where $\psi(t,x) \to \phi[r(t+m), \wedge(x+$ is topological automorphism.

VI. DIFFERENTIAL OPERATOR

The operator $\phi(t,x) \to D_{t,x}\phi()$ is defined on the space E into itself where $\phi(t,x) = K_\alpha(t,s-\eta,x,u-$

Proof: Let $D_{t,x}\phi(t,x) = \phi_1()$

$$\gamma_{E,l,q}\phi_1(t,x) = \sup_{t,x,\in I}\left|D_t^l D_x^q\ \phi_\alpha(t+m,x+n)\right|$$

$$= \sup_{t,x,\in I}\left|D_t^l D_x^q K_\alpha(t,s-\eta,x,u-\gamma)\right|$$

$$= \sup_{t,x,\in I}\left|D_t^{l+1}D_x^{q+1}C_{1\alpha}e^{i(s\tau+u\zeta)}e^{C_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)}\right.$$

$$= \left|C^2\sum_{a=0}^{l+1}\sum_{b=0}^{q+1}\frac{(l+1)!}{(l+1-2a)!}\frac{(q+1)!}{(q+1-2b)!}\frac{1}{a!}\frac{1}{b!}(iC_\alpha)^{l+q+2-(a+b)}2^{l+q+2-(a+b)}[t\cos\alpha-(s-\eta)]^{t+1-2a}\right.$$
$$\left.[x\cos\alpha-(u-\gamma)]^{q+1-2b}(\cos\alpha)^{a+b}e^{l+q+2-(a+b)}K_\alpha(t,s-\eta,x,u-\gamma)\right|$$

$<$ For any fixed $t,x \in$ and any fixed l, $0 < \infty \le$

Therefore $\phi_1(t,x) \in$,

$D_{t,x}\phi(t,x) \in E$

Hence proved.

VII. ADJOINT SHIFTING OPERATOR

The Adjoint shifting operators is a continuous function to .

The operator of (2) is $f(t,x) \to f(t+m, x+$ leads to operation transform formula

$$\left[F_{\alpha}^{\tau,\eta,\zeta,\gamma} f(t-m, x-n)\right] = i e^{C_{2\alpha}\left[(m^2+n^2)\cos\alpha - 2(m(s-\eta)+(u-\gamma)n)\right]} F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{e^{2i C_{2\alpha}\cos\alpha(mt+nx)} f(t,x)\}$$

Proof: Consider

$$F_{\alpha}^{\tau,\eta,\zeta,\gamma} f(t-m, x-n) = \langle f(t-m, x-n), \quad K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$$

$$= \langle f(t-m, x-n), \quad C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]}\rangle$$

$$= \langle f(t,x), \ C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((t+m)^2+(s-\eta)^2+(x+n)^2+(u-\gamma)^2+(x+n)^2\right)\cos\alpha - 2\left((t+m)(s-\eta)+(x+n)(u-\gamma)\right)\right]}\rangle$$

$$= \left\langle \begin{matrix} f(t,x), \quad e^{iC_{2\alpha}\left[\left(m(2t+m)+n(2x+n)\right)\cos\alpha - 2\left(m(s-\eta)+n(u-\gamma)\right)\right]} \\ C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]} \end{matrix} \right\rangle$$

$$= e^{iC_{2\alpha}\left[(m^2+n^2)\cos\alpha - 2(m(s-\eta)+n(u-\gamma))\right]} \langle f(t,x), \quad e^{iC_{2\alpha}2(tm+xn)\cos\alpha} K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$$

$$= e^{iC_{2\alpha}\left[(m^2+n^2)\cos\alpha - 2(m(s-\eta)+n(u-\gamma))\right]} \langle e^{2iC_{2\alpha}(tm+xn)\cos\alpha} f(t,x), \quad K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$$

$$= e^{iC_{2\alpha}\left[(m^2+n^2)\cos\alpha - 2(m(s-\eta)+n(u-\gamma))\right]} F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{e^{2iC_{2\alpha}(tm+xn)\cos\alpha} f(t,x)$$

Hence proved.

VIII. ADJOINT DIFFERNTIAL OPERATOR

The adjoint operator $f(t,x) \to D_{t,x} f(t,$ is continuous linear mapping from the dual space into itself corresponding transform formula is

$$F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{D_{t,x} f(t,x)\} = 4c\, F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{[t\cos\alpha - (s-\eta)][x\cos\alpha - (u-\gamma)] f(t,x)\}$$

Proof: $F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{D_{t,x} f(t,x)\} = \langle D_{t,x} f(t,x), \ K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$

$$= \langle f(t,x), \quad D_{t,x} K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$$

$$= \langle f(t,x), \quad -D_{t,x} C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]}\rangle$$

$$= \langle f(t,x), \quad -C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((s-\eta)^2+(u-\gamma)^2\right)\right]\cos\alpha} D_t e^{iC_{2\alpha}\left[t^2\cos\alpha - 2t(s-\eta)\right]} D_x e^{iC_{2\alpha}\left[x^2\cos\alpha - 2x(u-\gamma)\right]}\rangle$$

$$= \left\langle \begin{matrix} f(t,x), \quad 4(C_{2\alpha})^2[t\cos\alpha - (s-\eta)][x\cos\alpha - (u-\gamma)] \\ C_{1\alpha} e^{i(s\tau+u\zeta)} e^{iC_{2\alpha}\left[\left((s-\eta)^2+t^2+(u-\gamma)^2+x^2\right)\cos\alpha - 2\left((s-\eta)t+(u-\gamma)x\right)\right]} \end{matrix} \right\rangle$$

$$= \langle f(t,x), \quad 4(C_{2\alpha})^2[t\cos\alpha - (s-\eta)][x\cos\alpha - (u-\gamma)] K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle$$

$$= 4c\,\langle [t\cos\alpha - (s-\eta)][x\cos\alpha - (u-\gamma)] f(t,x), \quad K_{\alpha}(t, s-\eta, x, u-\gamma)\rangle, \qquad \text{where } (C_{2\alpha})^2 = c$$

$$= 4c\, F_{\alpha}^{\tau,\eta,\zeta,\gamma}\{[t\cos\alpha - (s-\eta)][x\cos\alpha - (u-\gamma)] f(t,x)\}$$

Hence proved.

IX. CONCLUSION

In the present work operators on the testing function space E and dual space E^* for Two-Dimensional Offset Fractional Fourier Transform are obtained.

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