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GENERALIZED EXPONENTIAL FUZZY INFORMATION MEASURES

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ABSTRACT

The present paper introduces two new parametric generalized exponential measure of fuzzy information with their proof of its validity. The particular case of proposed fuzzy information measure are also studied. Further, some more properties of the proposed information measure between different fuzzy sets are proved.

Keywords: Fuzzy set, Fuzziness, Fuzzy information Measures.

I INTRODUCTION

Entropy is one of the key measures of information first used by Shannon [1]. If X is a discrete random variable with probability distribution $P=(p_1, p_2,...p_n)$ in an experiment, then the information contained in this experiment is given by

$$H(P)=-\sum_{i=1}^{n} p_i \log p_i \qquad (1)$$

which is the well known Shannon [1] entropy.

The concept of fuzzy sets proposed by Zadeh [2] has proven useful in the context of pattern recognition, image processing, speech recognition, bioinformatics, fuzzy aircraft control, feature selection, decision making, etc.

Entropy as a measure of fuzziness was introduced by Zadeh [3]. Fuzzy entropy is an important concept for measuring fuzzy information. A measure of the fuzzy entropy of a fuzzy set is a measure of the fuzziness of the set.

During the last six decades, entropy, as a very important notion for measuring fuzziness degree or uncertain information in fuzzy set theory, has received a great attention. Fuzzy sets gained a vital attention from researchers for their application in various fields. De Luca et al. [4] introduced the measure of fuzzy entropy corresponding to Shannon [1] entropy. Later on Bhandari et al. [5] defined the exponential fuzzy entropy corresponding to Pal et al. [6] exponential entropy. Verma et al. [7] generalized the Pal et al. [6] exponential fuzzy entropy of order $\alpha > 0$.

Inspired by the above-mentioned work, we introduce a generalized methodology for measuring the information contained in a fuzzy set. We present two new parametric generalized exponential measure information and study the essential properties of these measure in order to check its authenticity.

The remainder of the paper is organized as follows. Section 2 is devoted to introduce some well-known concepts, and the notation related to fuzzy set theory and Fuzzy measures of information. In Section 3, we propose two new parametric generalized exponential measure of fuzzy information. Section 4 provides some properties of proposed parametric generalized exponential measure of fuzzy information. The final section concludes the paper.

II PRELIMINARIES

This section is devoted to introduce some well-known concepts and the notation. First of all, we will focus on the theory of fuzzy sets. Then, we will recall the axiomatic definition of a divergence measure for fuzzy sets.

2.1. Fuzzy Sets

Fuzzy sets are used to solve a lot of real world problems. Fuzziness, a feature of uncertainty, results from the lack of sharp difference of being or not being a member of the set, i.e., the boundaries of the set under consideration are not sharply defined.

A fuzzy set A defined on a universe of discourse X is given as Zadeh [2]:

$$A = \{\langle x, \mu_A(x)\rangle / x \in X\}$$

where $\mu_A : X \to [0,1]$ is the membership function of A. The membership value $\mu_A(x)$ describes the degree of the belongingness of $x \in X$ in A. When $\mu_A(x)$ is valued in {0, 1}, it is the characteristic function of a crisp (i.e., non-fuzzy) set. Zadeh [2] gave some notions related to fuzzy sets, some of them which we shall need in our discussion, are as follows:

(1) Compliment: $\bar{A}$ = Compliment of A $\Leftrightarrow$ $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ for all $x \in X$.

(2) Union: $A \cup B$ = Union of A and B $\Leftrightarrow \mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.

(3) Intersection: $A \cap B$ = Intersection of A and B $\Leftrightarrow \mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$ for all $x \in X$.

2.2. Fuzzy Measures of Information

In 1972 De Luca and Termini [4] introduced the measure of fuzzy entropy corresponding to Shannon [1] entropy given in (1) as

$$H(A) = -\sum_{i=1}^{n}[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))] \quad (2)$$

satisfying the following essential properties:

(P1) $H(A)$ is minimum if and only if A is a crisp set, i.e. $\mu_A(x_i) = 0$ or 1 for all x_i.

(P2) $H(A)$ is maximum if and only if A is most fuzzy set, i.e. $\mu_A(x_i) = 0.5$ for all x_i.

(P3) $H(A) = H(A^*)$, where A^* is sharpened version of A.

(P4) $H(A) = H(\bar{A})$, where $\bar{A}$ is the complement of A.

Later on Bhandari and Pal [5] defined the following exponential fuzzy entropy corresponding to Pal and Pal [6] exponential entropy as

$$E(A)=\frac{1}{n(\sqrt{e}-1)}\sum_{i=1}^{n}[\mu_A(x_i)e^{(1-\mu_A(x_i))}+(1-\mu_A(x_i))e^{\mu_A(x_i)}-1] \qquad (3)$$

Afterward, Tomar and Ohlan [8-12], Ohlan [12-21], Kaufmann [22], Ebanks [23], Kosko [24], Pal et al.[25] provided the non-parametric fuzzy entropy measures. In 1997, Kapur [37] initialized to present the generalized measures of fuzzy entropies that was further extended by Parkash [26], Hooda [27], Parkash and Sharma [28], Hooda and Bajaj [29], Hooda and Jain [30], Parkash and Gandhi [31], Verma and Sharma [32], Hooda and Jain [33], Kumar et al. [34], Kumar et al. [35], Hooda and Mishra [36] etc.

III TWO NEW PARAMETRIC GENERALIZED EXPONENTIAL MEASURE OF FUZZY INFORMATION

We now propose two new parametric generalized exponential measure of Information $E_\alpha^\beta(A)=\frac{1}{n(e^{(1-0.5^\alpha)^\beta}-1)}\sum_{i=1}^{n}[\mu_A(x_i)e^{(1-\mu_A^\alpha(x_i))^\beta}+(1-\mu_A(x_i))e^{(1-(1-\mu_A(x_i))^\alpha)^\beta}-1]$,

$\alpha>0,\ \beta>0$ (4)

$$E_{(\alpha,\beta)}(A)=\frac{1}{n(e^{(1-0.5^\alpha)}-1)}\sum_{i=1}^{n}[\mu_A^\beta(x_i)e^{(1-\mu_A^\alpha(x_i))}+(1-\mu_A(x_i))^\beta e^{(1-(1-\mu_A(x_i))^\alpha)}-1],\ \alpha>0,\ \beta>0 \qquad (5)$$

Theorem 3.1 The generalized fuzzy information measures $E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(A)$ are valid measures of fuzzy information.

Proof: P1 (Sharpness): The measures (4) and (5) clearly satisfy the property P1, i.e., $E_\alpha^\beta(A)=0$ and $E_{(\alpha,\beta)}(A)=0$ if and only if A is non-fuzzy set or crisp set.

P2 (Maximality): To verify that the proposed measure (4) is concave or maximal at $\mu_A(x_i)=0.5$;

Differentiating $E_\alpha^\beta(A)$ with respect to $\mu_A(x_i)$, we get

$$\frac{dE_\alpha^\beta(A)}{d\mu_A(x_i)}$$

$$=\frac{1}{n(e^{(1-0.5^\alpha)^\beta}-1)}\sum_{i=1}^{n}\left[\begin{array}{c}e^{(1-\mu_A^\alpha(x_i))^\beta}-\alpha\beta\mu_A^\alpha(x_i)(1-\mu_A^\alpha(x_i))^{\beta-1}e^{(1-\mu_A^\alpha(x_i))^\beta}-e^{(1-(1-\mu_A(x_i))^\alpha)^\beta}\\ +\alpha\beta(1-\mu_A(x_i))^{\alpha-1}[1-(1-\mu_A(x_i))^\alpha]^{\beta-1}e^{(1-(1-\mu_A(x_i))^\alpha)^\beta}\end{array}\right]$$

Let $0\le\mu_A(x_i)<0.5$, then

$$\frac{dE_\alpha^\beta(A)}{d\mu_A(x_i)}>0 \text{ for } 0<\alpha<1, 0<\beta<1 \text{ and } \alpha>1,\ \beta>1$$

and similarly, if $0.5 < \mu_A(x_i) \le 1$, then

$$\frac{dE_\alpha^\beta(A)}{d\mu_A(x_i)} > 0 \text{ for } 0 < \alpha < 1, 0 < \beta < 1 \; and \; \alpha > 1, \; \beta > 1.$$

and if $\mu_A(x_i) = 0.5$

$$\frac{dE_\alpha^\beta(A)}{d\mu_A(x_i)} = 0, \text{ for } 0 < \alpha < 1, 0 < \beta < 1 \; and \; \alpha > 1, \; \beta > 1.$$

Thus $E_\alpha^\beta(A)$ is a concave function which has a maximum value at $\mu_A(x_i) = 0.5$.

Now we prove the concavity of the proposed measure (5) or maximal at $\mu_A(x_i) = 0.5$;

Differentiating $E_{(\alpha,\beta)}(A)$ with respect to $\mu_A(x_i)$, we get

$$\frac{dE_{(\alpha,\beta)}(A)}{d\mu_A(x_i)} = \frac{1}{n(e^{(1-0.5^\alpha)} - 1)} \sum_{i=1}^{n} \left[\begin{array}{l} \beta\mu_A^{\beta-1}(x_i)e^{(1-\mu_A^\alpha(x_i))} - \alpha\mu_A^{\alpha+\beta-1}(x_i)e^{(1-\mu_A^\alpha(x_i))} \\ -\beta(1-\mu_A(x_i))^{\beta-1}e^{(1-(1-\mu_A(x_i))^\alpha)} + \alpha(1-\mu_A(x_i))^{\alpha+\beta-1}e^{(1-(1-\mu_A(x_i))^\alpha)} \end{array} \right]$$

Let $0 \le \mu_A(x_i) < 0.5$, then

$$\frac{dE_{(\alpha,\beta)}(A)}{d\mu_A(x_i)} > 0 \text{ for } 0 < \alpha < 1, 0 < \beta < 1 \; and \; \alpha > 1, \; \beta > 1$$

and similarly, if $0.5 < \mu_A(x_i) \le 1$, then

$$\frac{dE_{(\alpha,\beta)}(A)}{d\mu_A(x_i)} > 0 \text{ for } 0 < \alpha < 1, 0 < \beta < 1 \; and \; \alpha > 1, \; \beta > 1.$$

and if $\mu_A(x_i) = 0.5$

$$\frac{dE_{(\alpha,\beta)}(A)}{d\mu_A(x_i)} = 0, \text{ for } 0 < \alpha < 1, 0 < \beta < 1 \; and \; \alpha > 1, \; \beta > 1.$$

Thus $E_{(\alpha,\beta)}(A)$ is a concave function which has a maximum value at $\mu_A(x_i) = 0.5$.

Hence $E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(A)$ are maximum if and only if A is the most fuzzy set, i.e., $\mu_A(x_i) = 0.5$ for all i=1, 2, . . .,n.

P3 (Resolution): Since $E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(A)$ are increasing functions of $\mu_A(x_i)$ in the range $[0, 0.5)$ and decreasing function in the range $(0.5, 1]$, therefore

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$\mu_{A^*}(x_i) \le \mu_A(x_i) \Rightarrow E_\alpha^\beta(A^*) \le E_\alpha^\beta(A)$ in $[0, 0.5)$

and $\mu_{A^*}(x_i) \ge \mu_A(x_i) \Rightarrow E_\alpha^\beta(A^*) \ge E_\alpha^\beta(A)$ in $(0.5, 1]$.

Taking the above equations together, it comes

$$E_\alpha^\beta(A^*) \le E_\alpha^\beta(A).$$

Similarly, $E_{(\alpha,\beta)}(A^*) \le E_{(\alpha,\beta)}(A)$.

P4 (Symmetry): From the definition of $E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(A)$ and with $\mu_{\overline{A}}(x_i) = 1 - \mu_A(x_i)$, it is obvious that $E_\alpha^\beta(\overline{A}) = E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(\overline{A}) = E_{(\alpha,\beta)}(A)$.

Hence $E_\alpha^\beta(A)$ and $E_{(\alpha,\beta)}(A)$ satisfy all the four properties (P1) to (P4) of measures of fuzzy information, therefore these are valid measures of fuzzy information. The measures (4) and (5) can be called as the generalized fuzzy information measures.

In Particular for $\alpha = 1$ *and* $\beta = 1$, the measures (4) and (5) reduce to

$$E(A) = \frac{1}{n(\sqrt{e}-1)} \sum_{i=1}^{n} [\mu_A(x_i) e^{(1-\mu_A(x_i))} + (1-\mu_A(x_i)) e^{\mu_A(x_i)} - 1]$$

which is the measure (3) of Bhandari and Pal [1].

IV PROPERTIES OF TWO PARAMETRIC GENERALIZED EXPONENTIAL MEASURE OF FUZZY INFORMATION

Theorem 2: For $A, B \in FS(X)$, $E_\alpha^\beta(A \cup B) + E_\alpha^\beta(A \cap B) = E_\alpha^\beta(A) + E_\alpha^\beta(B)$.

Proof: Let $X_1 = \{x / x \in X, \mu_A(x_i) \ge \mu_B(x_i)\}$ (6)

$$X_2 = \{x / x \in X, \mu_A(x_i) < \mu_B(x_i)\} \quad (7)$$

Where $\mu_A(x_i)$ and $\mu_B(x_i)$ be the fuzzy membership functions of A and B respectively.

$$E_\alpha^\beta(A \cup B) = \frac{1}{n(e^{(1-0.5^\alpha)^\beta}-1)} \sum_{i=1}^{n} [\mu_{A\cup B}(x_i) e^{(1-\mu_{A\cup B}^\alpha(x_i))^\beta} + (1-\mu_{A\cup B}(x_i)) e^{(1-(1-\mu_{A\cup B}(x_i))^\alpha)^\beta} - 1]$$

$$= \frac{1}{n(e^{(1-0.5^\alpha)^\beta}-1)} \left[\begin{array}{l} \sum_{X_1} [\mu_A(x_i) e^{(1-\mu_A^\alpha(x_i))^\beta} + (1-\mu_A(x_i)) e^{(1-(1-\mu_A(x_i))^\alpha)^\beta} - 1] \\ + \sum_{X_2} [\mu_B(x_i) e^{(1-\mu_B^\alpha(x_i))^\beta} + (1-\mu_B(x_i)) e^{(1-(1-\mu_B(x_i))^\alpha)^\beta} - 1] \end{array} \right] \quad (8)$$

$$E_\alpha^\beta(A \cap B) = \frac{1}{n(e^{(1-0.5^\alpha)^\beta}-1)} \sum_{i=1}^{n} [\mu_{A\cap B}(x_i) e^{(1-\mu_{A\cap B}^\alpha(x_i))^\beta} + (1-\mu_{A\cap B}(x_i)) e^{(1-(1-\mu_{A\cap B}(x_i))^\alpha)^\beta} - 1]$$

$$=\frac{1}{n(e^{(1-0.5^{\alpha})^{\beta}}-1)}\left[\begin{array}{l}\sum_{X_1}[\mu_B(x_i)e^{(1-\mu_B^{\alpha}(x_i))^{\beta}}+(1-\mu_B(x_i))e^{(1-(1-\mu_B(x_i))^{\alpha})^{\beta}}-1] \\ +\sum_{X_2}[\mu_A(x_i)e^{(1-\mu_A^{\alpha}(x_i))^{\beta}}+(1-\mu_A(x_i))e^{(1-(1-\mu_A(x_i))^{\alpha})^{\beta}}-1]\end{array}\right] \quad (9)$$

Adding (8) and (9) gives

$E_{\alpha}^{\beta}(A\cup B)+E_{\alpha}^{\beta}(A\cap B)=E_{\alpha}^{\beta}(A)+E_{\alpha}^{\beta}(B)$.

Hence, the proof of theorem.

In Particular: For $A\in FS(X)$, $A\in FS(X)$ where $\overline{A}$ the complement of fuzzy set A, it gets

$$E_{\alpha}^{\beta}(A)=E_{\alpha}^{\beta}(\overline{A})=E_{\alpha}^{\beta}(A\cup\overline{A})=E_{\alpha}^{\beta}(A\cap\overline{A}) \quad (10)$$

Theorem 3: For $A,B\in FS(X)$, $E_{(\alpha,\beta)}(A\cup B)+E_{(\alpha,\beta)}(A\cap B)=E_{(\alpha,\beta)}(A)+E_{(\alpha,\beta)}(B)$.

Proof: Clearly the result can be proved on similar lines as in theorem 2.

In Particular: For $A\in FS(X)$, $A\in FS(X)$ where $\overline{A}$ the complement of fuzzy set A, it gets

$$E_{(\alpha,\beta)}(A)=E_{(\alpha,\beta)}(\overline{A})=E_{(\alpha,\beta)}(A\cup\overline{A})=E_{(\alpha,\beta)}(A\cap\overline{A}) \quad (11)$$

V CONCLUSION

In this paper, two new parametric generalizations of one of existing $R-$norm fuzzy information measures are proposed with the proof of their validity. The proposed generalized fuzzy measures of information are valid measures which reduce to the known measure on substituting the particular values of parameters. Some of the interesting properties of these measures have also been studied.

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