MULTI CRITERIA DECISION MAKING MODEL

Savitha M T[1], Dr. Mary George[2]

[1]Assistant Professor, [2]Associate Professor and Head, Department of Mathematics, Mar Baselios College of Engineering and Technology, Trivandrum

ABSTRACT

In this paper, we explain the value of both Trapezoidal and Triangular Fuzzy Numbers and develop a new ranking method based on the value of fuzzy number which in turn will be very helpful in decision making situations. Then we propose a Multi Criteria Decision Making (MCDM) Model based on the proposed ranking method. Arithmetic mean operation of fuzzy numbers is used for aggregating experts' judgments.

Keywords: Arithmetic mean, Multi Criteria Decision Making, Trapezoidal fuzzy number, Triangular fuzzy number, Value of fuzzy numbers,

I. INTRODUCTION

Most of the real life problems are complex in nature because of the indistinctness and impreciseness of the available data. In 1970 Bellman and Zadeh proposed the concept of fuzzy sets and fuzzy models to effectively handle these imprecise data which help us to avoid information loss through computing with words. To solve such real world problems, we can develop fuzzy expert systems by seeking the help of experts who have knowledge in that particular area. There may be many factors that influence a certain problem. While developing expert system, one has to rank these factors based on the experts' judgements. Usually experts' opinion is obtained as linguistic variables which can easily be converted into fuzzy numbers. For arriving at conclusions, we need to compile the experts' judgements. Subsequently, we use some ranking methods to find the order of these factors.

Ranking of fuzzy numbers plays a very significant role in linguistic multi-criteria decision making problems. Several fuzzy ranking methods have been proposed since 1976. The linguistic terms are represented quantitatively using fuzzy sets and then fuzzy optimal alternative is calculated which gives the relative merit of each alternative. S. Abbasbandy and T.Hajjari [1] in 2009 proposed a new method based on the left and right spreads at some α – levels and defined magnitude of fuzzy numbers. Ranking is done based on this magnitude. S. Abbasbandy and B. Asady [2] in 2006 proposed sign distance method by considering a fuzzy origin and then calculating distance with respect to the origin. If $u \in X$ and u_0 is the origin, then distance is defined as

$$D_p(u,u_0) = \left[\int_0^1 \left(\left|\underline{u}(r)\right|^p + |\overline{u}(r)|^p\right) dr\right]^{1/p},\ p \geq 1 .$$

F. Choobineh and Huishen Li [3] in 1993 proposed a new index for ranking without taking account of normality or convexity of fuzzy numbers. The new index for the fuzzy set A is defined as

$$R(a) = \frac{1}{2}\left[h_A - \frac{D(\mu_A,\mu_{U_A}) - D(\mu_A,\mu_{L_A})}{d-a}\right]$$

where h_A is the height of the fuzzy set A and μ_{U_A} and μ_{L_A} are the membership functions for the crisp barriers of the fuzzy set A. Ronald R.Yager [4] in 1981 proposed a ranking method using a function which is the integral of the mean of the level sets of the fuzzy subsets. The function F is defined from the subsets of unit interval I into I; for the fuzzy set A of I,

$F(A) = \int_0^{\alpha_{max}} M\, d\alpha$ where α_{max} is the maximum membership grade and M is the mean value of the members of an ordinary subset of the unit interval I.

Bass and H. Kwakernaak [5] proposed a method in 1977 consisting of computing weighted final ratings for each alternative and comparing the final weighted rating. J.F. Baldwin and NCF Guild [6] in 1979 improved the procedure proposed by Bass and Kwakernaak. They introduced a new method for pair wise comparison of all the alternatives instead of ranking them in the set of alternatives. This new method is helpful to determine how much one factor is greater than the other. Chung-Tsen Tsao 2002 [7] proposed a ranking method with the area between the centroid point and original point. J. Yao and K.Wu [8] in 2000 defined signed distance for ordering fuzzy numbers using decomposition principle. Signed distance between the fuzzy sets A and B is defined as

$$d(A,B) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha) - B_L(\alpha) - B_R(\alpha)]\, d\alpha$$

Lee and Li [9] in 1988 proposed a method based on the probability measures such as mean and standard deviation. Ching –Hsue Cheng in 1998 [10] proposed a method for ranking more than two fuzzy numbers simultaneously without considering the normality of fuzzy numbers. Ranking function is defined as the distance between centroid point and the original point. Based on the ranking function, ranking is determined. To improve Lee and Li's method, C.H Cheng proposed a ranking method also based on the coefficient of variation.

The Arithmetic mean operation of TrFNs is used to compile the variety of experts' judgements. Then a new ranking method based on the values of the fuzzy numbers is explained which is used to find the weights of the criteria involved in decision making process. A New MCDM model is also explained based on the computed scores of alternatives. This paper is organized as follows: Section 1 presents the preliminaries; Section 2 depicts the method of finding Arithmetic Mean of TrFNs; Section 3 describes the value based ranking method; Section 4 explains the new decision making model with illustrations and Section 5 concludes the work.

II. PRELIMINARIES

Definition 1.1

A fuzzy set A in a universe of discourse X is defined as the set of pairs,

$A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x) : X \to [0,1]$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition 1.2

The set $A(\alpha)$ is a convex set if $x, y \in A(\alpha) \Rightarrow \lambda x + (1-\lambda) y \in A(\alpha)$ where $\lambda \in [0,1]$.

Definition 1.3

For a fuzzy set A of X, the support of A, denoted by $supp(A)$ is the crisp subset of X which contains elements having nonzero membership grades in A.

$ie;\ supp\,(A) = \{x \in X : \mu_A(x) > 0\}$

Definition 1.4 [11]

A fuzzy number A is a fuzzy subset of the real line; $A: R \rightarrow [0,1]$ satisfying the following properties:

(i) A is normal (i.e. there exists $x_0 \in R$ such that $A(x_0) = 1$);

(ii) A is fuzzy convex ;

(iii) A is upper semi continuous on R. $ie;\quad \forall \varepsilon > 0,\ \exists\ \delta > 0$ such that

$$A(x) - A(x_0) < \varepsilon \text{ whenever } |x - x_0| < \delta;$$

(iv) The closure , $cl(supp\,(A))$ is compact.

Definition 1.5 [11]

The α-cut, $\alpha \in (0,1]$ of a fuzzy number A is a crisp set defined as $A(\alpha) = \{x \epsilon R : A(x) \geq \alpha\}$. Every A_α is a closed interval of the form $[A_L(\alpha), A_U(\alpha)]$.

Definition 1.6 [12]

A Trapezoidal fuzzy number denoted by A is defined as (l, m, n, u) where the membership function is given by

$$\mu_A(x) = \begin{cases} 0, & x \leq l \\ \dfrac{x-l}{m-l}, & l \leq x \leq m \\ 1, & m \leq x \leq n \\ \dfrac{u-x}{u-n}, & n \leq x \leq u \\ 0, & x \geq u \end{cases}$$

Definition 1.7 [11]

The value of a fuzzy number A is denoted and defined as

$$val(A) = \int_0^1 \alpha\left(A_U(\alpha) + A_L(\alpha)\right) d\alpha$$

III. ARITHMETIC MEAN OPERATION OF TRAPEZOIDAL FUZZY NUMBERS (TRFNS)

Consider the Trapezoidal Fuzzy Numbers:

$A_1 = (a_1, a_2, a_3, a_4)$, $A_2 = (b_1, b_2, b_3, b_4), \ldots,$ $A_n = (n_1, n_2, n_3, n_4)$ with membership functions,

$$\mu_{A_1}(x_1) = \begin{cases} 0\ , & x_1 \leq a_1 \\ \dfrac{x_1 - a_1}{a_2 - a_1}, & a_1 \leq x_1 \leq a_2 \\ 1\ , & a_2 \leq x_1 \leq a_3 \\ \dfrac{a_4 - x_1}{a_4 - a_3}, & a_3 \leq x_1 \leq a_4 \\ 0\ , & x_1 \geq a_4 \end{cases}$$

$$\mu_{A_2}(x_2) = \begin{cases} 0\ , & x_2 \leq b_1 \\ \dfrac{x_2 - b_1}{b_2 - b_1}, & b_1 \leq x_2 \leq b_2 \\ 1\ , & b_2 \leq x_2 \leq b_3 \\ \dfrac{b_4 - x_2}{b_4 - b_3}, & b_3 \leq x_2 \leq b_4 \\ 0\ , & x_2 \geq b_4 \end{cases}$$

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$$\mu_{A_n}(x_n) = \begin{cases} 0, & x_n \le n_1 \\ \frac{x_n - n_1}{n_2 - n_1}, & n_1 \le x_n \le n_2 \\ 1, & n_2 \le x_n \le n_3 \\ \frac{n_4 - x_n}{n_4 - n_3}, & n_3 \le x_n \le n_4 \\ 0, & x_n \ge n_4 \end{cases}$$

Or,

$$\mu_{A_1}(x_1) = max\left[min\left(\frac{x_1 - a_1}{a_2 - a_1}, 1, \frac{a_4 - x_1}{a_4 - a_3}\right), 0\right]$$

$$\mu_{A_2}(x_2) = max\left[min\left(\frac{x_2 - b_1}{b_2 - b_1}, 1, \frac{b_4 - x_2}{b_4 - b_3}\right), 0\right]$$

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$$\mu_{A_n}(x_n) = max\left[min\left(\frac{x_n - n_1}{n_2 - n_1}, 1, \frac{n_4 - x_n}{n_4 - n_3}\right), 0\right]$$

α- cuts of these fuzzy numbers are given by:

$$A_1(\alpha) = [A_{1L}(\alpha), A_{1U}(\alpha)] = [a_1 + \alpha(a_2 - a_1),\ a_4 - \alpha(a_4 - a_3)]$$

$$A_2(\alpha) = [A_{2L}(\alpha), A_{2U}(\alpha)] = [b_1 + \alpha(b_2 - b_1),\ b_4 - \alpha(b_4 - b_3)]$$

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$$A_n(\alpha) = [A_{nL}(\alpha), A_{nU}(\alpha)] = [n_1 + \alpha(n_2 - n_1),\ n_4 - \alpha(n_4 - n_3)]$$

Then we define the Arithmetic Mean of these fuzzy numbers as follows:

Let $A_V = \frac{A_1 + A_2 + \cdots + A_n}{n} = \left(\frac{a_1+b_1+\cdots+n_1}{n}, \frac{a_2+b_2+\cdots+n_2}{n}, \frac{a_3+b_3+\cdots+n_3}{n}, \frac{a_4+b_4+\cdots+n_4}{n}\right)$ with membership function,

$$\mu_{A_V}(X) = \begin{cases} sup\left[min\left(\frac{x_1 - a_1}{a_2 - a_1}, \frac{x_2 - b_1}{b_2 - b_1}, \dots, \frac{x_n - n_1}{n_2 - n_1}\right): \frac{x_1 + x_2 + \cdots + x_n}{n} = X\right] \\ \quad if \quad a_1 \le x_1 \le a_2, b_1 \le x_2 \le b_2, \dots, n_1 \le x_n \le n_2 \\ 1 \quad if \quad a_2 \le x_1 \le a_3, b_2 \le x_2 \le b_3, \dots, n_2 \le x_n \le n_3 \\ sup\left[min\left(\frac{a_4 - x_1}{a_4 - a_3}, \frac{b_4 - x_2}{b_4 - b_3}, \dots, \frac{n_4 - x_n}{n_4 - n_3}, \right): \frac{x_1 + x_2 + \cdots + x_n}{n} = X\right] \\ \quad if \quad a_3 \le x_1 \le a_4, b_3 \le x_2 \le b_4, \dots, n_3 \le x_n \le n_4 \end{cases}$$

That is,

$$\mu_{A_V}(X) = \begin{cases} \dfrac{X - \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right)}{\left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) - \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right)} & if \left(\frac{a_1 + b_1 + \cdots + n_1}{n}\right) \le X \le \left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) \\ 1 & if \left(\frac{a_2 + b_2 + \cdots + n_2}{n}\right) \le X \le \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right) \\ \dfrac{\left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) - X}{\left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) - \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right)} & if \left(\frac{a_3 + b_3 + \cdots + n_3}{n}\right) \le X \le \left(\frac{a_4 + b_4 + \cdots + n_4}{n}\right) \\ 0 & otherwise \end{cases}$$

The α- cut of A_V is:

$$A_V(\alpha) = [A_{VL}(\alpha),\ A_{VU}(\alpha)]$$
$$= \left[\left\{\left(\frac{a_1+b_1+\cdots+n_1}{n}\right)+\alpha\left(\frac{a_2+b_2+\cdots+n_2}{n}-\frac{a_1+b_1+\cdots+n_1}{n}\right)\right\},\right.$$
$$\left.\left\{\left(\frac{a_4+b_4+\cdots+n_4}{n}\right)-\alpha\left(\frac{a_4+b_4+\cdots+n_4}{n}-\frac{a_3+b_3+\cdots+n_3}{n}\right)\right\}\right]$$

Note: Similarly we can define arithmetic mean operation for triangular fuzzy numbers also.

IV. VALUE BASED RANKING METHOD

In section 4.1 value of fuzzy numbers is explained and in section 4.2 a new ranking method based on the value is given.

4.1 Value of Fuzzy Numbers

Proposition 1: The value of a Trapezoidal fuzzy number $A = (a, b, c, d)$ is given by

$$val(A) = \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proof:

The $\alpha - cut$ of the Trapezoidal fuzzy number $A = (a, b, c, d)$ is given by

$$A(\alpha) = [A_L(\alpha), A_U(\alpha)] = [a + \alpha(b - a),\ \ d - \alpha(d - c)]$$

Then value of the fuzzy number $A = (a, b, c, d)$ is denoted and defined by

$$val(A) = \int_0^1 \alpha\,[A_U(\alpha) + A_L(\alpha)]d\alpha$$
$$= \int_0^1 \alpha\,[d - \alpha(d - c) + a + \alpha(b - a)]d\alpha$$
$$= \frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$$

Proposition 2: The value of a Triangular fuzzy number $B = (a, b, c)$ is given by

$$val(B) = \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

Proof:

The $\alpha - cut$ of the Triangular fuzzy number $B = (a, b, c)$ is given by

$$B(\alpha) = [B_L(\alpha), B_U(\alpha)] = [a + \alpha(b - a),\ \ c + \alpha(b - c)]$$
$$val(B) = \int_0^1 \alpha\,[A_U(\alpha) + A_L(\alpha)]d\alpha$$
$$= \int_0^1 \alpha\,[c + \alpha(b - c) + a + \alpha(b - a)]d\alpha$$
$$= \frac{a}{6} + \frac{2b}{3} + \frac{c}{6}$$

4.2 Proposed Ranking Method

The significant step in decision making model is identifying factors and sub factors which are specific to the problem. Then the process involves obtaining an appropriate set of linguistic variables and associated fuzzy numbers in order to find the weights of the factors. These fuzzy numbers can be shown by membership functions. The selection of linguistic variables is chiefly a combination of knowledge elicitation and data preparation. We collect experts' judgments through questionnaires which are obtained as linguistic variables. These linguistic terms are fuzzified using the associated fuzzy numbers. We have to compile experts' judgements to have a group agreement of all experts. This step is done using arithmetic mean operation of fuzzy numbers which in turn result a single judgment fuzzy number for each of the factors. Then ranking of the factors can be done based on the values of these fuzzy numbers.

Using arithmetic mean operation we reach at a single judgment fuzzy number for the factor A_i as (l_i, m_i, n_i, u_i)

The value this fuzzy number corresponding to A_i , the i^{th} factor is given by

$$val(A_i) = \frac{l_i}{6} + \frac{m_i}{3} + \frac{n_i}{3} + \frac{u_i}{6}$$

Similarly we find the values corresponding to all factors. Based on these values we determine the ranking of A_i and A_j as follows:

(i) $val(A_i) > val(A_j) \Rightarrow A_i > A_j$

(ii) $val(A_i) < val(A_j) \Rightarrow A_i < A_j$

(iii) $val(A_i) = val(A_j) \Rightarrow A_i \sim A_j$

This method is used in the case of triangular fuzzy numbers also.

4.2.1 Pair wise comparison based on the proposed method.

Let $\{A_i, i = 1, 2, \dots, n\}$ be the set of all alternatives. Then a fuzzy relation is characterised by [13]

$$v(A_i, A_j) = \begin{cases} 1, & \text{if } A_i \text{ is absolutely preferred to } A_j \\ v \in (0.5, 1), & \text{if } A_i \text{ is absolutely preferred to } A_j \\ 0.5, & \text{not preferred to } A_j \\ v \in (0, 0.5), & \text{if } A_j \text{ is slightly preferred to } A_i \\ 0, & \text{if } A_j \text{ is absolutely preferred to } A_i \end{cases}$$

which helps for pair wise comparison.

Result 4.2.1

The weight of the alternative A_i over all other $(n-1)$ alternatives A_j is given by

$$weight(A_i) = \frac{\sum_{j=1, j\neq i}^{n} v(A_i, A_j)}{\sum_{i=1}^{n}\left(\sum_{j=1, j\neq i}^{n} v(A_i, A_j)\right)}$$

Note: (i) $0 \leq v(A_i, A_j) \leq 1$

(ii) $v(A_i, A_i) = 0.5$

(iii) $v(A_i, A_j) + v(A_j, A_i) = 1; i, j = 1, 2, \dots, n; i \neq j$

(iv) $\sum_{i=1}^{n} weight(A_i) = 1$

V. MULTI CRITERIA DECISION MAKING

We propose a Multi Criteria Decision Making Model for two types of decision making situations; Single decision maker & multiple decision makers. We discuss the model when the weights of the criteria are known. We define Score for each criteria based on the decision makers' judgments and weights of the criteria. Based on the score obtained we can reach at final conclusions.

5.1 Multi Criteria Decision Making Model- Single decision maker

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In this section we propose a simple and easiest method for multi criteria decision making when there is a single decision maker. Suppose we need to rank m alternatives $A_1, A_2, \ldots, A_m$ based on n criteria $C_1, C_2, \ldots, C_n$.

Let a_{ij} denotes the degree assigned by the decision maker that the alternative A_i satisfies criteria C_j . a_{ij} can be chosen based on some standard fuzzy scaling.

Let w_j be the weight of the criteria C_j for $j = 1, 2, \ldots, n$.

Then the Score of the alternative A_i to the criteria C_j is given by $Score(A_i \to C_j) = a_{ij}\, w_j$

Then the total Score of A_i based on all the criteria is denoted and defined by

$$Score(A_i) = \sum_{j=1}^{n} a_{ij} w_j$$

Procedure for Computation:

Step 1: First we calculate the weights of each criterion using the method explained in Section 3.2.

Step 2: Then we define linguistic terms for expressing decision makers' judgments and choose suitable scaling for these linguistic terms.

Step 3: Based on these judgments and scaling we assign degree of satisfaction for each criterion for each alternative as $a_{ij}; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.

Step 4: Then we calculate score for each alternative to each criterion as

$Score(A_i \to C_j) = a_{ij}\, w_j$

	C_1	C_2	. . .	C_n
A_1	$a_{11} w_1$	$a_{12} w_2$	. . .	$a_{1n} w_n$
A_2	$a_{21} w_1$	$a_{22} w_2$	. . .	$a_{2n} w_n$
.				
.				
A_m	$a_{m1} w_1$	$a_{m2} w_2$	. . .	$a_{mn} w_n$

Step 5: Total score of each alternative is calculated using $Score(A_i) = \sum_{j=1}^{n} a_{ij} w_j$ and using scores ranking of the alternatives is done.

Note: $Score(A_i) > Score(A_j) \Rightarrow A_i > A_j$

Example 5.1.1

Suppose we have to rank 5 alternatives $A_1, A_2, \ldots, A_5$ based on 3 criteria C_1, C_2 and C_3. Suppose we are using the linguistic terms very good, good, medium, poor and very poor.

Let us use the fuzzy scaling 4, 3, 2, 1 and 0 respectively for the above said linguistic terms. Assuming weights for the criteria as 0.29, 0.41 and 0.3 respectively, we can construct a decision table showing the score based on each criterion as follows:

	C_1	C_2	C_3
A_1	0.87	1.64	0.6
A_2	0	1.23	0.9
A_3	1.16	1.64	0.3
A_4	0.87	1.23	0.9
A_5	0	0.82	0.9

$Score(A_1) = 3.11$, $Score(A_2) = 2.013$, $Score(A_3) = 3.10$, $Score(A_4) = 3.00$, and $Score(A_5) = 1.72$

Hence we can rank the alternatives as $A_1 > A_3 > A_4 > A_2 > A_5$.

5.2 Multi Criteria Decision Making Model- Multiple decision makers

In this section we propose a simple and easiest method for multi criteria decision making when there are multiple decision makers. Suppose we need to rank m alternatives $A_1, A_2, \ldots, A_m$ based on n criteria $C_1, C_2, \ldots, C_n$. Let there are k decision makers.

Let a_{ijk} denotes the degree assigned by the k^{th} decision maker that the alternative A_i satisfies criteria C_j. a_{ijk} can be chosen based on some standard fuzzy scaling.

Let w_j be the weight of the criteria C_j for $j = 1, 2, \ldots, n$.

Let the weight of the decision makers $D_1, D_2, \ldots, D_k$ are $W_{D_1}, W_{D_2}, \ldots, W_{D_k}$

Each decision maker finds the score $Score(A_{ik})$ of each alternative A_i following the method explained in section 4.1 and then final score of the alternative A_i is obtained using the formula

$$S(A_i) = \sum_{k=1}^{k} [W_{D_k} * Score(A_{ik})]$$

5.2.1 Procedure for Computation

Step 1: Each decision maker follow the steps of section 4.1 (up to step 5) and obtain $Score(A_{ik})$. Then we obtain a decision matrix as follows.

	A_1	A_2	. . .	A_n
D_1	$Score(A_{11})$	$Score(A_{21})$	. . .	$Score(A_{n1})$
D_2	$Score(A_{21})$	$Score(A_{22})$	. . .	$Score(A_{2n})$
.				
.				
D_K	$Score(A_{k1})$	$Score(A_{k2})$	. . .	$Score(A_{kn})$

Step 2: If W_{D_i} be the weight of the decision maker D_i , then final score of the alternative A_j is calculated as

$$S(A_j) = \sum_{i=1}^{k} [W_{D_i} * Score(A_{ij})]$$

Example 5.2.1

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Suppose we have to rank 5 alternatives $A_1, A_2, \dots, A_5$ based on 3 criteria C_1, C_2 and C_3. Suppose we are using the linguistic terms very good, good, medium, poor and very poor. Let us assume that there are three decision makers D_1, D_2 and D_3 with weights 0.4, 0.25 and 0.35 respectively.

Let us use the scaling 4, 3, 2, 1 and 0 respectively for the above said linguistic terms. Assuming weights for the criteria as 0.29, 0.41 and 0.3 respectively, we can construct a decision matrix for each decision maker as follows:

Decision matrix-1

	C_1	C_2	C_3
A_1	0.87	1.64	0.6
A_2	0	1.23	0.9
A_3	1.16	1.64	0.3
A_4	0.87	1.23	0.9
A_5	0	0.82	0.9

Decision matrix-2

	C_1	C_2	C_3
A_1	1.16	1.64	0.9
A_2	0.29	1.23	1.2
A_3	1.16	1.23	0
A_4	1.16	1.23	1.2
A_5	0	0.82	0.9

Decision matrix-3

	C_1	C_2	C_3
A_1	0.58	1.64	0.9
A_2	0	1.64	0.9
A_3	0.87	1.23	1.2
A_4	0.87	1.64	0.9
A_5	0.29	0.82	0.6

Aggregating above decision matrices and obtaining the scores we get a single matrix as follows:

	D_1	D_2	D_3	Final Score, $S(A_j)$
A_1	3.11	3.7	3.12	3.26
A_2	2.013	2.72	2.54	2.38
A_3	3.1	2.39	3.3	2.99
A_4	3	3.59	3.41	3.29
A_5	1.72	1.72	1.71	1.72

Hence we can rank the alternatives as $A_1 > A_4 > A_3 > A_2 > A_5$.

VI. CONCLUSION

We have defined the values of different fuzzy numbers. Based on these values we proposed a simple ranking method for trapezoidal and triangular fuzzy numbers. The proposed method can successfully rank fuzzy numbers and their images. Hence it can be effectively used to calculate the weights of the attributes involving in decision making process. This new method does not require much computational effort in the ranking procedure and it depends only on the experts' judgments. The proposed decision model can be effectively used to rank alternatives when we have to take into consideration certain criteria while decision making. In this model we are considering the weights of decision makers since the proficiency, experience and knowledge of various decision makers' are not the same.

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