International Journal of Advanced Technology in Engineering and Science
Vol. No.5, Issue No. 03, March 2017
www.ijates.com
ijates
ISSN 2348 - 7550

FREE VIBRATION OF BEAM ON CONTINUOUS ELASTIC SUPPORT BY ANALYTICAL AND EXPERIMENTAL METHOD

Madhusmita Mohanta[1], Mallikarjun Jalageri [2], Gyan Setu[3]

[1] *Department of Mechanical Engineering, BIET, Mangalpally, Hyderabad 501 510, Telangana, (India)*
[2,3] *Department of Mechanical Engineering, K L University, Guntur 522 502, Andhra Pradesh (India)*

ABSTRACT

In this paper, free vibration of beam on an elastic foundation of Winkler type, which is distributed over a particular length of the beam, is considered. The governing differential equation of the beam issolved by using Newtonian method and further solved by variable separationmethod. The problem is handled by simply supported boundary conditions. Results are discussed in detail through comparison ofanalytical and experimental work. Ultimately, it is concluded that the results are in good agreement with each other.

Keywords: Winkler Foundation, Free Vibration, Elastic Foundation, Newtonian Method

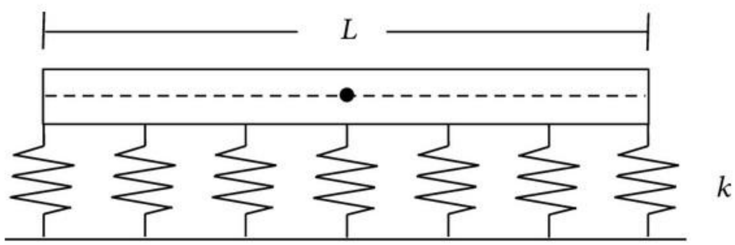
I INTRODUCTION

Dynamic analysis is an important part of structural investigation and the results of free vibration analysis are useful in this context. Knowledge of the natural frequency of vibration and associated mode shape is useful in vibration isolation problem. The problem of beams resting on elastic foundation is often encountered in the analysis of the foundations of buildings, highway and railroad structures, and of geotechnical structures in general. The simplest and most frequently employed mechanical model is that of Winkler. This model can be described in terms of a simple mathematical formulation which enables the finding of closed analytical solutions for problems of static and dynamic analysis. On the other hand, a complex foundation response may lead to unmanageable equations, which are difficult to solve numerically.

Free vibration analysis of beams has been extensively investigated by many researchers including Blevins [1] and Thambiratnam *et al.* [2]. Blevins has presented detailed information on the frequencies of vibration and associated mode shapes for practical use. Lai *et al.* [3] used a finite element method, based on the exact solution of the shape functions governing the end deflection of the beam, for studying the free vibration analysis of uniform beams on uniform elastic foundations. Though the theory of beams on elastic foundation has been in existence for decades, until recent times it has not been fully linked to the experimental method. Analytical solutions are available in the literature for beams on elastic foundation. Hetenyi [4] solved several problems of beams on elastic foundation subjected to static loads and elaborated on the application of the concept of beam on elastic foundation to analyze several other problems.

The work presented in this paper will first review a general analytical approach of analyzing the fundamental frequency of vibration of a simply supported beam resting on Winkler support and then the paper focuses on an experiment performed to validate the analytical approach. The paper concludes with a comparison of results from the experimental apparatus to the analytical method for the same boundary condition and elastic support.

II MATHEMATICAL FORMULATION



The equation of motion for Euler-Bernoulli beam resting on Winkler support is:

$$\frac{d^2}{dx^2}\left[EI(x)\frac{d^2y}{dx^2}\right]dx = -k_f y dx - \rho A(x)dx\frac{d^2y}{dt^2} \tag{1}$$

Where,

Restraining force for the element dx is $k_f ydx$,

And mass of beam per unit length is $\rho A(x)dx$ =m(x)

For prismatic beam (uniform cross-section),

$$EI\frac{d^4y}{dx^4} + k_f y = \rho A\frac{d^2y}{dt^2} \tag{2}$$

Assuming a product solution and performing the usual derivation and substitutions, the equation governing the modal function Y(x) is found to be,

$$\frac{d^4Y}{dx^4} + \left(\frac{-\rho A\omega^2}{EI} + \frac{k_f}{EI}\right)Y = 0 \tag{3}$$

Where, $\rho A = m$

With the notation,

$$\beta^4 = \frac{(\omega^2 m - k_f)}{EI} \tag{4}$$

Where, β is the foundation parameter.

Rewriting the solution in the standard form,

$$Y''''(x) - \beta^4 Y = 0 \tag{5}$$

With eigen solution,

$$Y(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 \cosh\beta x + C_4 \sinh\beta x \quad (6)$$

For the simply supported (hinged) beam, the boundary conditions are Y(0)= Y(L)= 0 and Y''(0)= Y''(L)=0.

$$\beta_r L = r\pi, \qquad r = 1,2,\dots..$$

Substituting the expression for β into this equation, we find the eigen values ω_r to be,

$$\omega_r = \left(\frac{r\pi}{L}\right)^2 \sqrt{\left[\frac{EI}{m} + \frac{k}{m}\left(\frac{L}{r\pi}\right)^4\right]}$$
(7)

III EXPERIMENTAL SETUP AND PROCEDURE

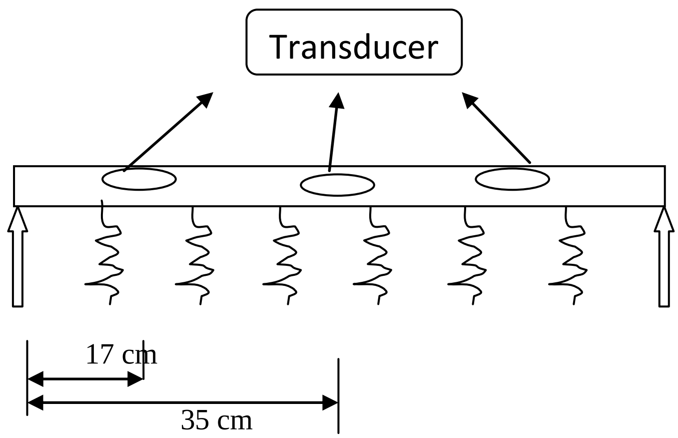


Figure1 Schematic Representation of Experimental Model

Figure2 Experimental Mode

Experimental setup for a simply supported beam on elastic foundation is shown in above figure. The beam of 70cm is divided into 7 stations and transducers are attached at the upper side of beam. Transducers are connected to the digital oscilloscope which shows the wave pattern generated on the screen. Three transducers are attached at equal

ijates
ISSN 2348 - 7550

distance of 17 cm from left end. Amplitude of vibration can be recorded from the oscilloscope. In simply supported beam amplitude of deflection at different stations is recorded by these three transducers at different station.

3.1 Experimental Setup Specifications

1	Length of beam, L	700 mm
2	Breadth of beam, B	50 mm
3	Depth of beam, W	5 mm
4	Young's modulus, E	200 GPa
5	Moment of inertia, I_X	520.83 mm^2
6	Spring stiffness, k	10 N/mm
7	Distance between two spring, l	100

IV RESULTS

S No	Experimental value	Analytical Value
1	28	27.33
2	42	42.70
3	92	92.90
4	188	188.30
5	217	216.16
6	242	240.85

Table1 Comparison Of Natural Frequency Values

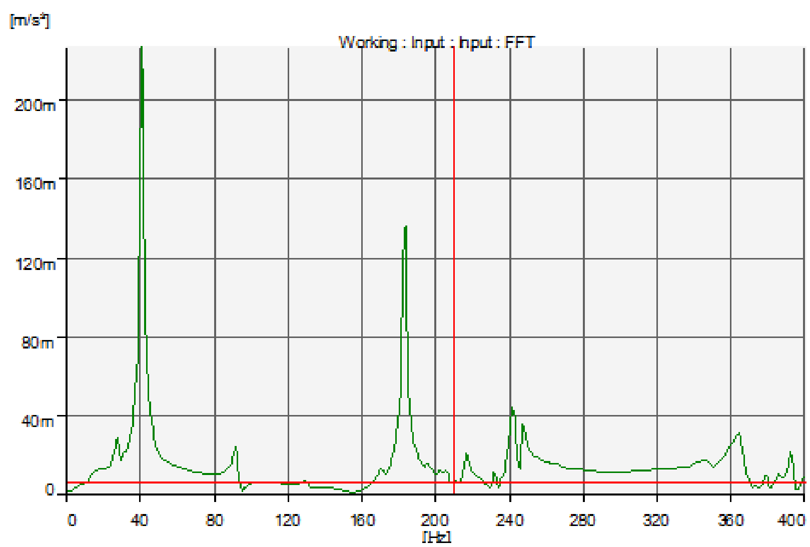


Figure3 Fourier Frequency Transform (FFT)

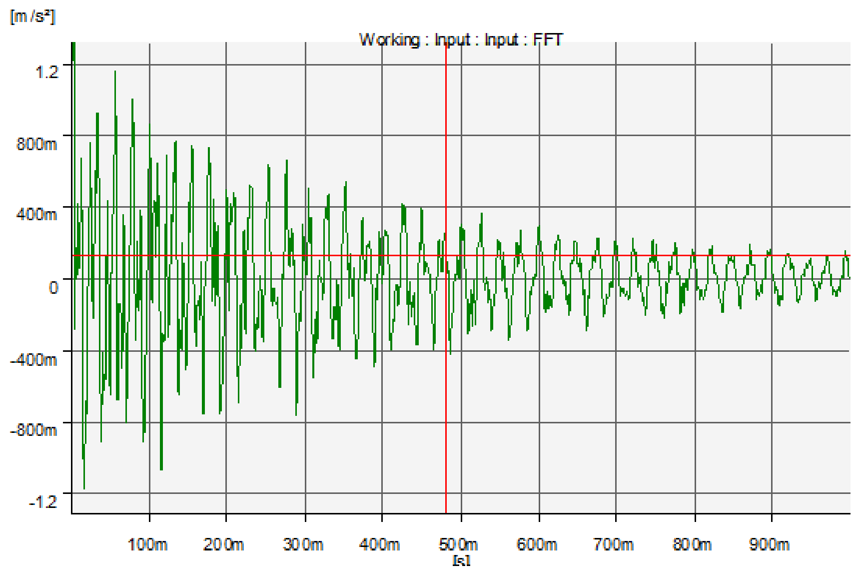


Figure4 Time Domain

Figure 3 shows the natural frequency for the beam on Winkler support and Figure 4 shows the decay rate. The experimental results for first few natural frequencies for different lengths of beam are being compared experimentally and analytically and it is found that to model continuous elastic support the limiting length of beam should be,

$$\text{Length of beam} \leq \frac{3\pi}{2\beta''} \quad , \qquad \text{Ref [5]}$$

Where, $\beta'' = \sqrt[4]{k/4EI}$

V CONCLUSION

The study covers the free vibration response of an Euler-Bernoulli beam on Winkler support and comparison of model by experimental and analytical approach. The elastic coefficient of the spring set is constant throughout the major axis of the beam. By solving the algebraic functions set, natural frequencies are obtained. The experimental results are found in good agreement with the analytical result. This analysis can be extended to treat dynamic loads and will find use in the earthquake response of beams on elastic foundation. This experiment was not intended to be highly accurate experimental evaluation but expected to demonstrate fundamental trends and behavior.

REFERENCES

1) R. D. Blevins, “Formulas for Natural Frequency and Mode Shape” Van Nostrand Reinhold, New York (1979).
2) D. Thambiratnam and Y. Zhuge, “Free vibration analysis of beams on elastic foundation”, Computers and Structures Vol 60 (1996).

3) Y. C. Lai, B. Y. Ting, W. S. Lee and W. R. Becker," Dynamic response of beams on elastic foundation" J. structure Engineering, ASCE 118, 853-858 (1992).

4) M. Hetenyi, "Beams on Elastic Foundation", University of Michigan Press, Ann Arbor, MI (1961).

5) A.P.Boresi, R.J.Schmidt, O.M.Sidebottom, "Advanced Mechanics of Materials", 5th edition,page-404-424(1993).