International Journal of Advanced Technology in Engineering and Science
Vol. No.5, Issue No. 01, January 2017
www.ijates.com
ijates
ISSN 2348 - 7550

BOUNDEDNESS OF IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS VIA LYAPUNOV FUNCTIONAL

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ABSTRACT

In this paper, the boundedness of the trivial solution for impulsive functional differential systems has been investigated using the Lyapunov functions and Razumikhin technique. The result obtained can be applied on finite delay or infinite delay impulsive systems. An example is given to illustrate the advantages of the theorem obtained.

Keywords: Impulsive Functional Differential Equation, Lyapunov functional, Uniformly bounded, Uniformly ultimate bounded

I. INTRODUCTION

Differential equations with impulsive effect provide mathematical models for many phenomena and processes in the field of natural sciences and technology. Recently a well developed stability theory of functional differential systems has come into existence [15-21]. Qualitative properties of impulsive differential equations have been intensively researched for years. In [3-11], by using lyapunov functions and Razumikhin techniques, some Razumikhin type theorems on stability and boundedness are obtained for a class of impulsive functional differential equations. In 1988, Wu [13] presented some sufficient conditions on stability andboundedness of neural functional differential equations with infinite delay using lyapunov functions.

The purpose of present paper is to investigate the stability and boundedness theorems for functional differential equations with infinite delays.

II. PRELIMINARIES

Consider the system of impulsive functional differential equations with infinite delays

$$x'(t) = F\big(t, x(.)\big), \qquad t > t^* \tag{1}$$

$$x(t_k) = J_k\big(x(t_k{}^-)\big), \qquad k = 1,2, \dots \dots \tag{2}$$

Where $t > t^* \geq \alpha \geq -\infty$. Let $F(t, x(s); \alpha \leq s \leq t)$ or $F(t, x(.))$ be a volterra type functional, its values are in R^n. $x'(t)$ denote the right hand derivative of $x(t)$, $t^* < t_k < t_{k+1}$ with $t_k \to \infty$, $J_k: R^n \to R^n$ and $x(t_k^-) = \lim_{t \to t_{k-0}} x(t)$. For any $t \geq t^*$, $PC([\alpha, t], R^n)$ will be written as $PC(t)$. Define $PCB(t) = \{x \in PC(t): x$ is bounded$\}$. For any $\emptyset \in PCB(t)$, the norm of $\emptyset$ is defined by $\|\emptyset\| = \|\emptyset\|^{[\alpha,t]} = \sup_{\alpha \leq s \leq t} |\emptyset(s)|$. For any $\sigma \geq t^*$ and $\emptyset \in PCB(\sigma)$, the equations (1) and (2), one associates an initial condition of the form

$x(t) = \emptyset(t), \qquad \alpha \leq t \leq \sigma.$

It is shown in [12] that under the following hypothesis (H_1) – (H_4), the initial value problem (1) and (2) has unique solution $x(t, \sigma, \emptyset)$.

(H_1) F is continuous on $[t_{k-1}, t_k) \times PC(t)$ for $k = 1,2, \ldots$.. where $t_0 = t^*$.For all $\varphi \in PC(t)$ and $k = 1,2, \ldots$.., the limit $\lim_{(t,\emptyset) \to (t_k^-, \varphi)} F(t, \emptyset) = F(t_k^-, \varphi)$ exists.

(H_2) F is Lipchitz in $\emptyset$ in each compact set in $PCB_h(t)$. More precisely, for any $\gamma \in [\alpha, \sigma + \beta)$ and every compact set $G \subset PCB_h(t)$, there exists a constant $L = L(\gamma, G)$ such that $|F(t, \varphi(.)) - F(t, \psi(.))| \leq L\|\varphi - \psi\|^{[\alpha,t]}$ whenever $t \in [\alpha, \gamma]$ and $\varphi, \psi \in G$

(H_3) The functions $(t, x) \to I(t, x)$ is continuous on $[t^*, \infty) \times R^n$.For any $\rho > 0$, there exist a $\rho_1 > 0$ $(0 < \rho_1 < \rho)$ such that $x \in S(\rho_1)$ implies that $x + I(t_k, x) \in S(\rho)$ for $k \in Z^+$

(H_4) For any $x(t) \in PC([\alpha, \infty), R^n), F(t, x(.)) \in PC([t^*, \infty), R^n)$

In this paper, in order to study the boundedness of (1) and (2), it is assumed that $F(t, 0) \equiv 0$, $J_k(0) \equiv 0$ so that $x(t) \equiv 0$ is a solution of (1) and (2), called the zero solution. Also throughout the paper, it is assumed that $\beta = \infty$. Consider the solution $x(t, \sigma, \varphi)$ of equations (1) and (2) which can be continue to ∞ from the right side of σ.

Definition 2.1 The solutions of (1) and (2) are said to be

(S_1) Uniformly bounded (UB), if for each number $B_1 > 0$, there exist a number B_2 such that $[\sigma \geq t^*, \varphi \in PCB_{B_1}(\sigma)$ and $t \geq \sigma]$ implies that $|x(t, \sigma, \varphi)| \leq B_2$.

(S_2) Uniformly Ultimate bounded (UUB) with the bound B if for each $B_3 > 0$, there exist a $T > 0$ such that $[\sigma \geq t^*, \varphi \in PCB_{B_3}(\sigma)$ and $t \geq \sigma + T]$ implies that $|x(t, \sigma, \varphi)| \leq B$

Definition 2.2 A function $V(t, x): [\alpha, \infty) \times R^n \to R^n$ belongs to class v_0 if

(A_1) V is continuous on each of the sets $[t_{k-1}, t_k) \times R^n$ and for all $x \in R^n$ and $R \in Z^+$, the limits $\lim_{(t,\ y) \to (t_k^-,\ x)} V(t, y) = V(t_k^-, x)$ exists.

(A_2) V is locally lipschitzian in x and $V(t, 0) \equiv 0$.

Definition 2.3 A functional $V(t, \emptyset): [\alpha, \infty) \times PCB(t) \to R^n$ belongs to class $v_0(.)$ if

(B_1) V is continuous on each of the sets $[t_{k-1}, t_k) \times PCB(t)$ and for all $\varphi \in PCB(t)$ and $k \in Z^+$, the limits $\lim_{(t,\emptyset) \to (t_k^-,\ \varphi)} V(t, \emptyset) = V(t_k^-, \varphi)$ exists.

(B_2) V is locally lipschitzian in $\emptyset$ and $V(t, 0) \equiv 0$.

Definition 2.4 A functional $V(t, \emptyset)$ belongs to class $v_0^*(.)$ if $V \in v_0(.)$ and for any $x \in PC([\alpha, \infty), R^n)$, $V(t, x(.))$ is continuous for $t \geq t^*$.

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ISSN 2348 - 7550

Let $V \in v_0$, for any $(t,x) \in [t_{k-1}, t_k) \times R^n$, the right hand derivative $V'(t,x(t))$ along the solution $x(t)$ of (1) and (2) is defined by

$$V'(t,x(t)) = \lim_{h \to 0^+} sup \frac{V(t+h, x(t+h)) - V(t,x(t))}{h}$$

We say that $W: [0,\infty) \to [0,\infty)$ belongs to class $\mathfrak{R}$ if W is continuous and strictly increasing and satisfies $W(0) = 0$

We say a function $\eta : [t^*, \infty) \to [0, \infty)$belongs to class PIM if η is measurable function such that for any $l > 0$ and every $\epsilon > 0$, there exist a $T \geq t^*$ and $\delta > 0$ such that $[t \geq T, Q \subset [t-l, t]$, an open set , and $\mu(Q) \geq \epsilon]$ imples $\int_Q \eta(t)dt \geq \delta$ where $\mu(Q)$ is a measure of set Q.

Lemma 1: Assume that x is a piecewise right continuous bounded function and $\eta \in PIM$. Then for any $W_4, W_5 \in \mathfrak{R}$, any $h > 0$ and each $\lambda > 0$, there exist $\mu > 0$ such that $\int_{t-h}^{t} W_4(|x(s)|)ds \geq \lambda$ implies $\int_{t-h}^{t} W_5(|x(s)|)ds \geq \mu$

Lemma 2: Assume that x is a piecewise right continuous bounded function and $\eta \in PIM$. Then for any $W_4, W_5 \in \mathfrak{R}$, any $h > 0$ and each $\lambda > 0$, there exist $\mu > 0$ and $T \geq t^* + h$ such that $t \geq T$, $\int_{t-h}^{t} W_4(|x(s)|)ds \geq \lambda$ implies$\int_{t-h}^{t} W_5(|x(s)|)\eta(s)ds \geq \mu$

II. MAIN RESULTS

Theorem 3.1: Let $W_i \in \mathfrak{R}(i = 1,2,3), \tau > 0$ be a constant, $V_1(t,x) \in v_0$, $V_2(t,\emptyset) \in {v_0}^*(.)$, $\Phi \in C(R^+, R^+), \Phi \in L^1[0,\infty), \Phi(t) \leq \mathcal{K}$ and $q \in C(R^+, R^+)$ such that $q(s)$ is non increasing,$q(s) > 0, s > 0$. Assume that he following conditions hold:

(i) $W_1(|\emptyset(t)|) \leq V(t,\emptyset(.)) \leq W_2(|\emptyset(t)|) + W_3 \int_{\alpha}^{t} \Phi(\mathrm{t-s}) W_4(|\emptyset(s)|)ds$, where $V(t,\emptyset(.)) = V_1(t,\emptyset(t)) + V_2(t,\emptyset(.)) \in v_0(.)$;

(ii) For each $k \in Z^+$ and all $x \in R_n$, $V(t_k, J_k(x)) \leq (1+b_k)V({t_k}^-, x)$, where $b_k \geq 0$ with $\sum_{k=1}^{\infty} b_k < \infty$

(iii) There exist a number $U > 0$ such that for any $L > 0$, there is a $h > 0$ such that $V'(t,x(t)) \leq -W_5(|x(t)|)ift \geq \sigma, \|x\|^{[\alpha,t]} \leq L$, $|x(t)| \geq U$and$P\left(V(t,x(t))\right) > V(s,x(s))$ for $\max\{\alpha, t-h\} \leq s \leq t$ where $P\in C(R^+,R^+), P(s) > Ms$ for $s > 0, M = \prod_{k=1}^{\infty}(1+b_k)$, $x(t) = x(t,\sigma,\varphi)$ is the solution of (1) and (2).

Then the solution of (1) and (2) is UB and UUB.

Proof: First we show UB. Let $B_1 \geq U$be given such that $MW_2(|x(t)|) \leq W_1(B_2)$ and $MW_3(JW_4(|x(t)|)) \leq W_1(B_2)$where $J = \int_0^{\infty} \Phi(u)du$.Let $\sigma \geq t^*, \varphi \in PCB_{B_1}(\sigma)$ and $x(t) = x(t,\sigma,\varphi)$ be the solution of (1) and (2)

Set $V_1(t) = V_1(t,x(t)), V_2(t) = V_2(t,x(.))$And $V(t) = V_1(t) + V_2(t)$.

Then

$V(t) \leq W_2(|x(t)|) + W_3(JW_4(|x(t)|)) \leq W_2(B_1) + W_3(JW_4(B_1)) = M^{-1}W_1(B_2)$ $,\alpha \leq t \leq \sigma$

Let $\sigma \in [t_{m-1}, t_m)$ for some $m \in Z^+$. Suppose that there is a $\hat{t} \in (\sigma, t_m)$ such that

$V(\hat{t}) = M^{-1}W_1(B_2),\ V'(\hat{t}) \geq 0,\ \ V(t) < V(\hat{t}),\ t \in [\sigma, \hat{t})$.

Thus

$$P(V(\hat{t})) > MV(\hat{t}) = W_1(B_2) \geq V(s),\ \ \alpha \leq s \leq \hat{t} \tag{3}$$

Let $L = W_1^{-1}(M^{-1}W_1(B_2))$, then $\|x\|^{[\alpha,\hat{t}]} \leq L$. By (3), for$h > 0$

$$P(V(\hat{t})) > V(s), \qquad for \max\{\alpha, \hat{t} - h\} \leq s \leq \hat{t}$$

On the other hand

$W_2(|x(\hat{t})|) \geq V(\hat{t}) = M^{-1}W_1(B_2)$ $=W_2(B_1)$.

Thus $(|x(\hat{t})|) \geq B_1 \geq U$. By assumption (iii),

$$V'(\hat{t}) \leq -W_5(|x(\hat{t})|) < 0$$

It is a contradiction and so we have

$$V(t) \leq M^{-1}W_1(B_2),\quad \sigma \leq t \leq t_m \tag{4}$$

By assumption (ii), we have

$$V(t_m) = V_1(t_m) + V_2(t_m) \leq (1 + b_m)V_1(t_m{}^{-1}) + (1 + b_m)V_2(t_m)$$
$$= (1 + b_m)V(t_m{}^{-1}) \leq M^{-1}(1 + b_m)W_1(B_2)$$

Similarly we can prove that

$$V(t) \leq M^{-1}(1 + b_m)W_1(B_2), \qquad t_m \leq t \leq t_{m+1}$$
$$V(t_m) \leq M^{-1}(1 + b_m)(1 + b_{m+1})W_1(B_2)$$

By induction, one can prove in general that

$$V(t) \leq M^{-1}(1 + b_m) \ldots\ldots\ldots\ldots..(1 + b_{m+i})W_1(B_2), \qquad t_{m+i} \leq t \leq t_{m+i+1}, \quad i = 0,1, \ldots\ldots\ldots$$

$V(t_{m+i+1}) \leq M^{-1}(1 + b_m) \ldots\ldots\ldots\ldots..(1 + b_{m+i+1})W_1(B_2)$

Therefore, we have

$W_1(|x(t)|) \leq V(t) \leq W_1(B_2),\ \ t \geq \sigma$

This proves UB.

Next we will prove UUB.

Let $B = W_1^{-1}(MW_2(U)) + W_1^{-1}(MW_3J(W_4))$ such that $MW_2(|x(t)|) \leq W_1(B_4)$ and $MW_3(JW_4(|x(t)|)) \leq W_1(B_4)$.For any $B_3 \geq U$ be given. By the preceding argument, one could find a$B_4 > B$ such that $\sigma \geq t^*$, $\varphi \in PCB_{B_3}(\sigma)$ imply $V(t) \leq W_1(B_4)$ and $(|x(t)|) \leq B_4$, $t \geq \sigma$.

Since $\Phi \in L^1[0, \infty)$, it follows that there is a $r > 1$ such that

$$W_4(B_4) \int_r^{\infty} \Phi(\mathrm{u})\mathrm{du} < \frac{1}{2} W_3{}^{-1}\left(\frac{1}{2M} W_1(B_2)\right)$$

And

$$W_4{}^{-1}\left[\frac{1}{2Kr} W_3{}^{-1}\left(\frac{1}{2M} W_1(B_2)\right)\right] \leq W_2{}^{-1}\left(\frac{1}{2M} W_1(B_2)\right) \tag{5}$$

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ISSN 2348 - 7550

It is clear that for $t \geq \sigma + r$,

$$W_1(|x(t)|) \leq V(t) \leq W_2(|x(t)|) + W_3\left[\int_{\alpha}^{t} \Phi(\mathrm{t-s}) W_4(|x(s)|)ds\right]$$

$\leq W_2(|x(t)|) + W_2\left[\frac{1}{2} W_3^{-1}\left(\frac{1}{2M} W_1(B_2)\right) + K \int_{t-r}^{t} W_4(|x(s)|)ds\right]$ (6)

Let $0 < d < \inf\{P(s) - Ms: M^{-1}W_1(B) \leq s \leq W_1(B_4)\}$ and N be the first positive integer such that $W_1(B) + Nd \geq MW_1(B_4)$. For $L = B_4 > 0$, let $h > 0$ be the corresponding number in assumption (iii). Set

$$\tau_i = \sigma + i\left[h + \frac{(1 + M^*)W_1(B_4)}{W_5(U)}\right], \qquad i = 0,1, \dots \dots, N,$$

Where $M^* = \sum_{k=1}^{\infty} b_k$.

We will prove that

$$V(t) \leq W_1(B) + (N-1)d, \qquad t \geq \tau_i, \qquad i = 0,1, \dots, N \qquad (7)_i$$

Clearly $(7)_0$ holds. Now suppose, $(7)_i$ holds for some $0 \leq i < N$. We prove that

$$V(t) \leq W_1(B) + (N-i-1)d, \qquad t \geq \tau_{i+1}, \qquad i = 0,1, \dots, N \qquad (7)_{i+1}$$

We first claim that there exist a

$\bar{t} \in I_i = [\tau_i + h, \tau_{i+1}]$ such that

$$V(\bar{t}) \leq M^{-1}[W_1(B) + (N-i-1)d] \qquad (8)$$

Suppose for all $t \in I_i$, $V(t) > M^{-1}[W_1(B) + (N-i-1)d]$

Then $W_2(U) + W_3\big(JW_4(U)\big) = M^{-1}W_1(B) < V(t) \leq W_1(B_4)$ and so

$$\|x\|^{[\alpha,t]} \leq L, \qquad |x(t)| \geq U,$$

$$P\big(V(t)\big) > MV(t) + d > W_1(B) + (N-1)d \geq V(s) \text{ for } \max\{\alpha, t-h\} \leq s \leq t$$

By assumption (iii), one has for $t \in I_i$, $V'(t) \leq -W_5(|x(t)|)$,

Thus for $t \in I_i$,

$$V(t) \leq V(\tau_i + h) - \int_{\tau_i+h}^{t} W_5(|x(s)|)ds + \sum_{\tau_i+h \leq t_k < t} [V(t_k) - V(t_k^{-})]$$

$$\leq W_1(B_4) - W_5(U)(t - \tau_i - h) + \sum_{k=1}^{\infty} b_k V(t_k^{-})$$

$$\leq (1 + M^*)W_1(B_4) - W_5(U)(t - \tau_i - h)$$

On the other hand, from (6), one has for $t \in I_i$

$$W_2(|x(t)|) + W_3\left[\frac{1}{2} W_3^{-1}\left(\frac{1}{2M} W_1(B_2)\right) + K \int_{t-r}^{t} W_4(|x(s)|)\, ds\right]$$

$$\geq V(t) > M^{-1}[W_1(B_2) + (W-i-1)d] \geq M^{-1}W_1(B_2)$$

We claim that for $t \in [\tau_i + h + 2r, \tau_{i+1}]$,

$W_3\left[\frac{1}{2} W_3^{-1}\left(\frac{1}{2M} W_1(B_2)\right) + K \int_{t-2r}^{t} W_4(|x(s)|)\, ds\right] \geq \frac{1}{2M} W_1(B_2)$ (9)

In fact, if $W_2(|x(t)|) \leq \frac{1}{2}M^{-1}W_1(B_2)$, it is clear that (9) holds. In the case when

$W_2(|x(t)|) > \frac{1}{2}M^{-1}W_1(B_2)$, if (9) does not holds, then we have by (5),

$$K_r W_4\left({W_2}^{-1}\left(\frac{1}{2M}W_1(B_2)\right)\right) < 0$$

Which is a contradiction and so (5) holds.

From (9), we have

$$\int_{t-2r}^{t} W_4(|x(s)|)\,ds \geq \frac{1}{2K}{W_3}^{-1}\left(\frac{1}{2M}W_1(B_2)\right), \qquad \tau_i + h + 2r \leq t \leq \tau_{i+1}$$

By lemma 1, there exist a $\mu > 0$ such that

$$\int_{t-2r}^{t} W_5(|x(s)|)\,ds \geq \mu, \quad \tau_i + h + 2r \leq t \leq \tau_{i+1}$$

Let $t = \tau_{i+1}$, we have

$$V(\tau_{i+1}) \leq (1+M^*)W_1B_4 - \frac{W_5U(1+M^*W_1B_4)}{W_5(U)} = 0$$

Which is a contradiction and so (8) holds.

Let $l = \inf \quad \{k \in Z^+ : t_k > \bar{t}\}$. We claim that

$$V(t) \leq M^{-1}[W_1(B) + (N-i-1)d], \quad \bar{t} \leq t < t_l \qquad (10)$$

Otherwise, there is a $\hat{t} \in (\bar{t}, t_l)$ such that

$$V(\hat{t}) > M^{-1}[W_1(B) + (N-i-1)d] \geq V(\bar{t})$$

This implies that there is a $\tilde{t} \in (\bar{t}, \hat{t})$ such that

$$V(\tilde{t}) = M^{-1}[W_1(B) + (N-i-1)d] \geq V(t), \quad t \in (\bar{t}, \hat{t})$$

$$V'(\tilde{t}) > 0$$

Thus $P\big(V(\tilde{t})\big) > MV(\tilde{t}) + d \geq V(s)$ for $\max\{\alpha, \tilde{t} - h\} \leq s \leq \tilde{t}$ and $\|x\|^{[\alpha,\tilde{t}]} \leq L$, $|x(\tilde{t})| > U$

By assumption (iii)

$$V'(\tilde{t}) \leq -W_5|x(\tilde{t})| < 0$$

This is a contradiction. So (10) holds.

From (10) and assumption (ii) we have

$$V(t_l) \leq (1+b_l)V({t_l}^-) \leq M^{-1}(1+b_l)[W_1(B) + (N-i-1)d]$$

Similarly, one can prove in general, that for $j = 0,1, \ldots ....$

$$V(t) \leq M^{-1}(1+b_l) \ldots \ldots \ldots (1+b_{l+i+1})[W_1(B) + (N-i-1)d], \quad t_{l+i} \leq t \leq t_{l+i+1},$$

$$V(t_{l+i+1}) \leq M^{-1}(1+b_l) \ldots \ldots \ldots (1+b_{l+i})[W_1(B) + (N-i-1)d]$$

Thus,

$$V(t) \leq W_1(B) + (N-i-1)d, \quad t \geq \bar{t}$$

Therefore, $(7)_{i+1}$ holds. By the induction, $(7)_i$ hold for all $i = 0,1,2, \ldots \ldots \ldots, N$. Thus, when $i = N$, we have

$$W_1(|x(t)|) \le V(t) \le W_1(B), \quad t \ge \sigma + N\left(h + \frac{(1+M^*)W_1(B_4)}{W_5(U)}\right)$$

Finally set

$$T = N\left(h + \frac{(1+M^*)W_1(B_4)}{W_3(U)}\right),$$

Then

$|x(t)| \le B$ fort $\ge \sigma + T$.

This completes the proof.

Theorem 3.2: Instead of assumption (iii) in Theorem 3.1, we impose (iii)'

For any $t \ge \sigma$, there is a $\eta \in PIM$ such that for any $L > 0$, there is a $h > 0$ such that

$V'(t, x(t)) \le -\eta(t)W_5(|x(t)|)if\|x\|^{[\alpha,t]} \le L$, and $P\left(V(t,x(t))\right) > V(s, x(s))$ for $\max\{\alpha, t-h\} \le s \le t$

where P$\in C(R^+, R^+), P(s) > Ms$ for $s > 0$, $M = \prod_{k=1}^{\infty}(1+b_k)$, $x(t) = x(t,\sigma,\varphi)$ is the solution of (1) and (2).

Then the solution of (1) and (2) is UB and UUB.

Remark: The above Theorem 3.2 is generalization of Theorem 3.1. Since the proof is similar as that of Theorem 3.1, we omit it here for the sake of brevity.

Example: Consider the equation

$$x'(t) = -a(t)x(t) + \int_{-\infty}^{t} c(t-s)x(s)ds + f(t), \quad t \ge 0 \tag{11}$$

$$x(t_k) = (1+b_k)x(t_k{}^-), \quad k \in Z^+ \tag{12}$$

Where a, c and f are continuous functions, $c(t) \in L^1[0,\infty)$, $|f(t)| \le H$ for some $H > 0$, with $b_k \ge 0$ and $\sum_{k=1}^{\infty} b_k < \infty$

Suppose

$$-a(t) + M\int_0^{\infty} |c(u)|du \le -\lambda$$

where$\lambda > 0$ and $M = \prod_{k=1}^{\infty}(1+b_k)$

Then the zero solution of (11) and (12) is UB and UUB.

Let $V_1 \in v_0$, $V_2 \in v_0{}^*(.)$as $V_1(t, \emptyset(0)) = |\emptyset(0)|$,

$$V_2(t,\emptyset) = \int_{-\infty}^{t}\int_{t}^{\infty} |c(u-s)|du|\emptyset(s)|ds$$

Then $V_1(t_k, J_k(x)) = |J_k(x)| \le (1+b_k)|x| = (1+b_k)V_1(t_k{}^-, x)$

For any solution $x(t) = x(t,\sigma,\varphi)$ of (11),

$$V'(t, x(t)) \le -L|x(t)|$$

By Theorem 3.1, the zero solution of (11) and (12) is UB and UUB.

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