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THE STACKELBERG MODEL OF DUOPOLY–AN ECONOMIC ANALYSIS AND APPLICATIONS

Shivani Sanan[1], Dr. Amanpreet Singh[2]

[1]*Research Scholar, Punjab Technical University Jalandhar, Punjab, (India)*

[2] *Post Graduate, Department of Mathematics SGTB Khalsa College, Anandpur Sahib (Ropar) (India)*

ABSTRACT

The objective of this paper is to economically analyse the Stackelberg Duopoly Model and applications of Stackelberg Security Games (SSG). The Stackelberg model is amongst the most frequently applied models of oligopolistic interaction. It refers to the situation in which one firm, the Stackelberg leader can commit to its output first. A sequential order of move is today's interpretation of Stackelberg model. Stackelberg markets yield higher outputs, higher consumer rents and higher welfare to the society.

I. INTRODUCTION

In 1934, Heinrich Frieher Stackelberg in his 'Market Structure and Equilibrium' proposed a strategic game theory in which the firms choose quantities sequentially rather than simultaneously as in Cournot's model (1838). The first to choose their output is the Stackelberg leader while the second firm is Stackelberg follower. As the industry leader, first firm i.e. the leader firm is able to implement its decision before its rival firms. After both the firms have chosen their outputs, the total output is sold at the prevailing market price. Stackelberg model is also known as 2- period model. In the first period, the leader chooses its quantity. This decision is irreversible and cannot be altered in the second period. In the 2nd period, the follower chooses its quantity after observing the quantity chosen by the leader. There is also an important assumption of perfect information in Stackelberg game. If a proper leader- follower relationship is established and the follower behaved as assumed, then two Stackelberg equilibria will be added to Cournot equilibria. Here, the first section discusses the model setup. The second section states the economic analysis of the model and welfare to the society. The third part compares the model with Bertrand's and Cournot's models. The concluding part states the applications of the model.

II. MODEL SETUP

Let two firms produce quantities p and q respectively of a given good i.e.

$$Q = p + q$$

Let P(Q) be the price function which tells us how much each unit of quantity is to sell.

$P(Q|Q < \lambda) = \lambda - Q$, where the parameter λ captures demand

$$P(Q|Q > \lambda) = 0$$

Now we require a cost function of each item. Assume that cost to produce each item is c. Further assume that c is less than λ. This assumption means that there are some levels of production which are profitable. Thus, cost functions for the two firms

$$C_1(p) = cp$$
$$C_2(q) = cq$$

The final thing we need to do is write out utility functions for each firm. Assuming that this is a normal form game then we can write our firm 1 and2's utility functions as following

$$U_1(p,q) = p(P(Q) - C_1(p))$$
$$U_2(p,q) = q(P(Q) - C_2(q))$$

Since we have values of P(Q) and C_i, we can rewrite the utility functions as

$$U_1(p,q) = p(\lambda - p - q - c)$$
$$U_2(p,q) = q(\lambda - p - q - c)$$

Since the firm moves sequentially, second firm's move will decide the move of first firm's after that. So we take utility function of second firm and find the maximum value of q.

$$\frac{\partial U_2}{\partial q} = \lambda - p - 2q - c$$

For maximum value of q , we put $\frac{\partial U_2}{\partial q} = 0$ we get $\lambda - p - 2q - c = 0$

$$q* = \frac{\lambda - p - c}{2}$$

which is same as we find equilibrium in Cournot's model. From here the process is changed and it is sequential and we must return to the firm 1's utility function.

$$U_1(p,q) = p(\lambda - p - q - c)$$
$$= p\left(\lambda - p - \frac{\lambda - p - c}{2} - c\right)$$
$$= p\left(\frac{\lambda}{2} - \frac{p}{2} - \frac{c}{2}\right)$$
$$Thus, \quad U_1(p,q) = p\ \frac{\lambda}{2} - \frac{p^2}{2} - \frac{pc}{2}$$

Now we find the maximum value of p, put $\frac{\partial U_1}{\partial p} = 0$, we get

$$\frac{\lambda}{2} - p - \frac{c}{2} = 0 \qquad \Rightarrow p* = \frac{\lambda - c}{2}$$

This gives us the value of p which the firm 1 will produce in equilibrium. From this value we can find the value of q also:

$$q* = \frac{\lambda - c}{4}$$

III. ECONOMIC ANALYSIS- WELFARE TO THE SOCIETY

In a leader-follower game, the leader typically produces a larger quantity and makes larger profits whereas the follower produces a lower quantity and makes lower profits under a symmetric cost assumption. Strategic substitutability among firm's quantity decisions is the main driving force behind this result. Therefore as we move from simultaneous move games to sequential move ones, this reallocation of firms' output decisions will increase total output and consumer's total welfare. The other effect that drives our result is the non- strategic transmission effect. In a leader-follower game, the follower is more informed about demand when taking their decision. Therefore, under Stackelberg competition, the followers are likely to produce more when demand is high and produce less when it is low. This implies that prices are less responsive to the underlying demand

shock under Stackelberg competition. This greater price stability induces higher welfare which is beneficial to the society.

IV. COMPARISON WITH OTHER OLIGOPOLY MODELS

The aggregate Stackelberg output is greater than the aggregate Cournot output, but less than the aggregate Bertrand output .

In terms of economic efficiency, the Stackelberg's result is quite similar to Cournot's result. The Nash equilibrium is not *Pareto efficient* which leads to loss in economic efficiency. Nevertheless, the loss is lower in the Stackelberg duopoly than n Cournot's model.

The Stackelberg price is lower than the Cournot price, but greater than the Bertrand price.

Daughety considers a parameterized class of Stackelberg markets and shows that all sequential move structures are beneficial compared to simultaneous move Cournot markets.

The Stackelberg consumer surplus is greater than the Cournot consumer surplus, but lower than Bertrand consumer surplus.

Stackelberg markets yield higher consumer rents and higher welfare levels than Cournot's markets.

The aggregate Stackelberg output is greater than pure monopoly or cartel, but less than the perfectly competitive output.

The Stackelberg price is lower than the pure monopoly or cartel price, but greater than the perfectly competitive price.

V. APPLICATIONS

An application of Stackelberg leardership is Microsoft's dominance in software markets. Although Microsoft can make decisions first, other smaller companies react to Microsoft's actions when making their own decisions. The actions of these followers, in turn, affect Microsoft.

Another example of Stackelberg model is the sequentiality of games in crude oil market. There, Saudi Arabia acts as a Stackelberg leader and all other members of OPEC (Organisation of Petroleum Exporting Countries) act as followers.

The global mobile phone DRAM market which is the main memory component of most computers and many electronic systems can be considered as oligopoly market with sequentially ordered firms competing based on their quantities under full commitment. DRAM's that have similar memory capacity and that are produced by different vendors can be considered as highly substitutable goods. The biggest vendors such as Samsung, Hynix, and Micron take the lead. In the second mover group are Nanya, Etron, Powerchip. The second mover group typically make their investment after they observe the investment decisions of the biggest vendors in the first group.

The chemical by-produts of the tanning process in the leather industries are sent to the nearby rivers and waterways. As a result, the Ganga River and many others have become extremely contaminated, leading to substantial health problems. The Stackelberg Security Games (SSG) focus on the problem of pollution prevention by modelling the interaction between the inspection agency (the leader) and leather tanneries (many followers).

VI. CONCLUSION

Stackelberg model is amongst the frequently applied models of today's oligopolistic market structures. Actual markets exhibit such a sequential order of moves. In the Stackelberg duopoly model, one firm determines its profit maximizing quantity and other firm reacts to that quantity. Stackelberg's modification to the Cournot model is important because it represents situations in which firms are not identical. Thus an important implication of the Stackelberg model is that it improves market efficiency.

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