International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 08, August 2016
www.ijates.com
ijates
ISSN 2348 - 7550

Cube difference labeling of some helm and wheel related graphs

G. Amuda[1], S. Meena[2]

1. Lecturer in Mathematics Government College for Women (A), Kumbakonam, Tamil Nadu, India.

2. Associate Professor and Head Department of Mathematics Government Arts College,. C. Mutlur, Chidambaram, Tamil Nadu, India.

ABSTRACT

A Let G be a (p,q) graph. G is said to admit a cube difference labeling if there exists an injection $f:V(G)\rightarrow\{0,1,2,...,p-1\}$ such that the edges set of G has assigned a weight defined by the absolute cube difference of its end-vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph.In this chapter we investigate the existence of cube difference labeling of some helm and wheel related graphs, such as switching of rim vertices of wheel, apex vertex in helm, duplication of vertices of wheel and helm.

KEYWORDS - Cube difference labeling , Cube difference graph.

I. INTRODUCTION

All graphs in this paper are simple finite undirected and nontrivial graph G = (V, E) with vertex set V and the edge set E. For graph theoretic terminology, we refer to Harary [2]. A dynamic survey on graph labeling is regularly updated by Gallian [3] and it is published by Electronic Journal of Combinatorics. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. The square difference labeling is previously defined by V. Ajitha, S. Arumugam and K. A. Germina [1]. The concept of cube difference labeling was first introduced by J. Shiama and it was proved in [7] that many standard graphs like Pn, Cn, complete graphs, ladder, lattice grids, wheels, comb, star graphs, crown, dragon, coconut trees and shell graphs are cube difference Labeling. Andalso some references [4]-[6].Some graphs like cycle cactus graph, special tree and a New Key graph[8] can also be investigated for the cube difference.

Definition: 1.1: Let $G = (V, E)$ be a graph. A difference labeling of G is an injection f from V to the set of non-negative integer with weight function $f*$ on E given by $f*(uv) = |f(u) - f(v)|$ for every uv edge in G.A graph with a difference labeling defined on it is called a labeled graph.

Definition: 1.2: Let $G = (V, E)$ be a graph .G is said to be cube difference labeling if there exist a bijection $f: V(G) \rightarrow \{0,1,2,....p-1\}$ such that the induced function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = \left|[f(u)]^3 - [f(v)]^3\right|$ is injective.

Definition: 1.3: A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G, removing all the edges to v and adding edges joining v to every other vertex which are not adjacent to v in G.

Definition: 1.4: A helm, $H_n, n \geq 3$ is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the wheel's rim.

Definition 1.5: The wheel graph W_n **is** join of the graphs C_n and K_1. i.e. $W_n = C_n + K_1$. Here vertices corresponding to C_n are called rim vertices and C_n is called rim of *n W* while the vertex corresponds to K_1 is called apex vertex.

Definition 1.6: Duplication of a vertex v_k of a graph *G* produces a new graph G_1 by adding a vertex $v_k^{'}$ with $N(v_k^{'}) = N(v_k)$. In other words a vertex $v_k^{'}$ is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to $v_k^{'}$ also Duplication of a vertex v_k by a new edge $e = v_k^{'}v_k^{''}$ in a graph *G* produces a new graph $G^{'}$ such that $N(v_k^{'}) = \{v_k, v_k^{''}\}$ and $Nv_k^{''} = \{v_k, v_k^{'}\}$.

Definition 1.7: Duplication of an edge $e = uv$ of a graph *G* produces a new graph *G*' by adding an edge $e' = u'v'$ such that $N(u') = N(u) \cup (v') - \{v\}$ and $N(v') = N(v) \cup (u') - \{u\}$.

Duplication of an edge $e = uv$ **by a new vertex** *w* in a graph *G* produces a new graph *G*' such that $N(w) = \{u, v\}$.

II. MAIN RESULTS

Theorem: 2.1. For $n > 3$, the graph obtained by switching any rim vertex of wheel W_n admits a cube difference labeling.

Proof:

Let u be a apex vertex and let $u_1, u_2, \ldots, u_n$ be rim vertices of W_n and G_{u_1} be the graph obtained by switching a rim vertex u_1 of W_n.

Here $\left|V\left(G_{u_1}\right)\right| = n+1$ and $\left|E\left(G_{u_1}\right)\right| = 3n-5$.

Define a vertex labeling $f : E\left(G_{u_1}\right) \to \{0,1,2,\ldots,n\}$ as follows.

$$f(u_i) = i \qquad \text{for } 1 \le i \le n$$

$$f(u) = 0$$

We have to prove that the induced edge labeling function $f^* : E\left(G_{u_1}\right) \to N$ defined by

$f^*(uv) = \left| f(u)^3 - f(v)^3 \right|$ for all $uv \in E(G)$ is injective.

The edge labelings are

$$f^*(u_i u_{i+1}) = \left| i^3 - (i+1)^3 \right| \qquad \text{for } 2 \le i \le n-1$$

$$= \{19, 37, \ldots, 3n^2 - 3n + 1\}$$

$$f^*(u_1 u_{i+1}) = \left| 1 - (i+1)^3 \right| \qquad \text{for } 2 \le i \le n-2$$

$$= \{26, 63, \ldots, (n-1)^3 - 1\}$$

$$f^*(uu_i) = i^3 \qquad \text{for } 2 \le i \le n$$

$$= \{8, 27, \ldots, n^3\}$$

Thus all the edge labels are distinct. Hence the graph obtained by switching any rim vertex of wheel graph W_n admits a cube difference labeling.

Theorem: 2.2. The graph obtained by switching the apex vertex of helm H_n admits a cube difference labeling.

Proof:

Let u be a apex vertex and let $u_1, u_2, \ldots, u_n$ be the rim vertices and $v_1, v_2, \ldots, v_n$ be the pendant vertices of helm H_n.

Let G_u be the graph obtained by switching the apex vertex of helm H_n.

Here $\left|V\left(G_u\right)\right| = 2n+1$ and $\left|E\left(G_u\right)\right| = 3n$.

Define a vertex labeling $f : E\left(G_u\right) \to \{0,1,\ldots,2n\}$ as follows

$$f\left(u_i\right) = i \qquad \text{for } 1 \le i \le n$$

$$f\left(v_i\right) = n+i \qquad \text{for } 1 \le i \le n$$

$$f\left(u\right) = 0$$

We have to prove that the induced edge labeling function $f^* : E\left(G_u\right) \to N$ defined by

$f^*\left(uv\right) = \left|f\left(u\right)^3 - f\left(v\right)^3\right|$ for all $uv \in E\left(G\right)$ is injective.

The edge labelings are

$$f^*\left(u_i u_{i+1}\right) = \left|i^3 - \left(i+1\right)^3\right| \qquad \text{for } 1 \le i \le n-1$$

$$= \{7,19,\ldots,3n^2 - 3n + 1\}$$

$$f^*\left(u_n u_1\right) = n^3 - 1$$

$$f^*\left(uu_i\right) = i^3 \qquad \text{for } 1 \le i \le n$$

$$= \{1,8,\ldots,n^3\}$$

$$f^*\left(uv_i\right) = \left(n+i\right)^3 \qquad \text{for } 1 \le i \le n$$

$$= \{\left(n+1\right)^3, \left(n+2\right)^3, \ldots, \left(2n\right)^3\}$$

$$f^*\left(u_i v_i\right) = \left|i^3 - \left(n+i\right)^3\right| \qquad \text{for } 1 \le i \le n$$

$$= \{\left(n+1\right)^3 - 1, \left(n+2\right)^3 - 2^3, \ldots, 7n^3\}$$

Thus all the edge labels are distinct. Hence the graph obtained by switching the apex vertex of helm H_n admits a cube difference labeling.

Theorem: 2.3. The graph G obtained by duplicating all the vertices of the wheel graph. W_n, except the apex vertex admits a cube difference labeling.

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 08, August 2016
www.ijates.com
ijates
ISSN 2348 - 7550

Proof:

Let G be the graph obtained by duplicating all the vertices of wheel graph W_n except the apex vertex. Let $u_1^{'}, u_2^{'}, \ldots u_n^{'}$ be the new vertices of G by duplicating $u_1, u_2, \ldots, u_n$. Then $V(G) = \{c, u_i, u_i^{'} / 1 \le i \le n\}$ and

$$E(G) = \{cu_i, cu_i^{'} / 1 \le i \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i u_{i+1}^{'}, u_{i+1} u_i^{'} / 1 \le i \le n-1\}$$

$$\cup \{u_1 u_n, u_1^{'} u_n, u_n^{'} u_1\}$$

Define a vertex labeling $f : V(G) \to \{0, 1, 2 \ldots 2n\}$ as follows

$$f(u_i) = 2i - 2 \qquad \text{for } 1 \le i \le n$$

$$f(c) = 2n$$

$$f(u_i^{'}) = 2i - 1 \qquad \text{for } 1 \le i \le n$$

We have to prove that the induced edge labeling function $f^* : E(G) \to N$ defined by

$f^*(uv) = \left| f(u)^3 - f(v)^3 \right|$ for every $uv \in E(G)$ is injective.

The edge labelings are

$$f^*(u_i u_{i+1}) = \left| (2i-2)^3 - (2i)^3 \right| \qquad \text{for } 1 \le i \le n-1$$

$$= \{8, 56, \ldots, 24n^2 - 72n + 56\}$$

$$f^*(u_n u_1) = (2n-2)^3$$

$$f^*(cu_i) = \left| (2n)^3 - (2i-2)^3 \right| \qquad \text{for } 1 \le i \le n$$

$$= \{(2n)^3, (2n)^3 - 2^3, \ldots, (2n)^3 - (2n-2)^3\}$$

$$f^*(cu_i^{'}) = \left| (2n)^3 - (2i-1)^3 \right| \qquad \text{for } 1 \le i \le n$$

$$= \{8n^3 - 1^3, 8n^3 - 3^3 \ldots, 12n^2 - 6n + 1\}$$

$$f^*(u_i u_{i+1}^{'}) = \left| (2i-2)^3 - (2i+1)^3 \right| \qquad \text{for } 1 \le i \le n-1$$

$$= \{27, 117, \ldots, 36n^2 - 90n + 63\}$$

International Journal of Advanced Technology in Engineering and Science
Vol. No.4, Issue No. 08, August 2016
www.ijates.com
ijates
ISSN 2348 - 7550

$$f^*(u_n u_1') = (2n-2)^3 - 1$$

$$f^*(u_i' u_{i+1}) = \left|(2i-1)^3 - (2i)^3\right| \quad \text{for } 1 \le i \le n-1$$

$$= \{7, 37, \ldots, 12n^2 - 30n + 19\}$$

$$f^*(u_n' u_1) = (2n-1)^3$$

Thus all the edge labels are distinct. Hence the graph obtained by duplicating all the vertices of the wheel graph W_n except the apex vertex admits a cube difference labeling.

Theorem: 2.4. The graph obtained by duplicating all the vertices in the rim of helm H_n admits a cube difference labeling.

Proof:

Let $G = H_n$ be the graph with vertex set $V(G) = \{c, u_i, v_i / 1 \le i \le n\}$ and the edge set $E(G) = \{cu_i, u_i v_i / 1 \le i \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u, u_n\}$.

Let G' be the graph obtained by duplicating all the rim vertices in H_n and let the new vertices be $u_1', u_2' \ldots u_n'$, then the vertex set $V(G') = \{c, u_i, v_i, u_i' / 1 \le i \le n\}$ and the edge set

$$E(G') = \{cu_i, cu_i', u_i v_i, v_i u_i' / 1 \le i \le n\} \cup \{u_i u_{i+1}, u_i u'_{i+1}, u_i' u_{i+1} / 1 \le i \le n-1\}$$
$$\cup \{u_n u_1, u_n u_1', u_n' u_1\}$$

Here $|V(G')| = 3n+1$ and $|E(G')| = 7n$.

Define a vertex labeling $f : V(G') \to \{0, 1, 2 \ldots 3n\}$ as follows

$$f(u_i) = i - 1 \quad \text{for } 1 \le i \le n$$

$$f(v_i) = n + 2i - 1 \quad \text{for } 1 \le i \le n$$

$$f(u_i^1) = n + 2i \quad \text{for } 1 \le i \le n$$

$$f(c) = n$$

We have to prove that the induced edge labeling function $f^* : E(G') \to N$ defined by

$f^*(uv) = \left|f(u)^3 - f(v)^3\right|$ for every $uv \in E(G)$ is injective.

The edge labelings are

$$f^*(cu_i) = \left|n^3 - (i-1)^3\right| \quad \text{for } 1 \le i \le n$$

$$= \left\{n^3, n^3 - 1, \ldots, 3n^2 - 3n + 1\right\}$$

$$f^*(u_i v_i) = \left|(i-1)^3 - (n+2i-1)^3\right| \quad \text{for } 1 \le i \le n$$

$$= \left\{(n+1)^3, (n+3)^3 - 1, \ldots, 26n^3 - 24n^3 + 6n\right\}$$

$$f^*(u_i u_{i+1}) = \left|(i-1)^3 - i^3\right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{1, 7, \ldots, 3n^2 - 9n + 7\right\}$$

$$f^*(u_n u_1) = (n-1)^3$$

$$f^*(cu_i^{'}) = \left|n^3 - (n+2i)^3\right| \quad \text{for } 1 \le i \le n$$

$$= \left\{(n+2)^3 - n^3, (n+4)^3 - n^3, \ldots, 26n^3\right\}$$

$$f^*(v_i u_i^{'}) = \left|(n+2i-1)^3 - (n+2i)^3\right| \quad \text{for } 1 \le i \le n$$

$$= \left\{(n+2)^3 - (n+1)^3, (n+4)^3 - (n+3)^3, \ldots, (3n)^3 - (3n-1)^3\right\}$$

$$f^*(u_i u_{i+1}^{'}) = \left|(i-1)^3 - (n+2i+2)^3\right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{(n+4)^3, (n+6)^3 - 1, \ldots, (3n)^3 - (n-2)^3\right\}$$

$$f^*(u_n u_1^{'}) = 9n^2 + 9n + 9$$

$$f^*(u_{i+1} u_i^{'}) = \left|(n+2i)^3 - i^3\right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{(n+2)^3 - 1, (n+4)^3 - 2^3, \ldots, (3n-2)^3 - n^3\right\}$$

$$f^*(u_n^{'} u_1) = (3n)^3$$

Thus the edge labels are distinct. Hence the graph obtained by duplicating all the rim vertices of Helm

ijates
ISSN 2348 - 7550

H_n admits a cube difference labeling.

Theorem: 2.5. The graph G obtained by duplicating all the vertices of the helm H_n, except the apex vertex admits a cube difference labeling.

Proof:

Let $G = H_n$ be the graph with vertex set $V(G) = \{c, u_i, v_i / 1 \le i \le n\}$ and the edge set $E(G) = \{cu_i, u_i v_i / 1 \le i \le n\} \cup \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_1 u_n\}$.

Let G' be the graph obtained by duplicating all the vertices in the helm H_n, except the apex vertex c.

Let $u_1', u_2' ... u_n'$ and $v_1', v_2' ... v_n'$ be the new vertices of G by duplicating $u_1, u_2 ... u_n$ and $v_1, v_2 ... v_n$ then the vertex set $V(G') = \{c, u_i, v_i, u_i', v_i' / 1 \le i \le n\}$ and the edge set

$$E(G') = \{cu_i, cu_i', u_i v_i, v_i u_i', u_i v_i' / 1 \le i \le n\} \cup \{u_i u_{i+1}, u_i u_{i+1}', u_{i+1} u_i / 1 \le i \le n-1\}$$

$$\cup \{u_n u_1, u_n u_1', u_n' u_1\}$$

Here $|V(G')| = 4n+1$ and $|E(G')| = 8n$.

Define a vertex labeling $f : V(G') \to \{0, 1, 2 ... 4n\}$ as follows

$$f(c) = n$$

$$f(u_i) = i-1 \quad \text{for } 1 \le i \le n$$

$$f(v_i) = n+i \quad \text{for } 1 \le i \le n$$

$$f(u_i') = n+2i+2 \quad \text{for } 1 \le i \le n$$

$$f(v_i') = n+2i+3 \quad \text{for } 1 \le i \le n$$

We have to prove that the induced edge labeling function $f^* : E(G') \to N$ defined by

$f^*(uv) = \left| f(u)^3 - f(v)^3 \right|$ for every $uv \in E(G)$ is injective.

The edge labelings are

$$f^*(cu_i) = \left| n^3 - (i-1)^3 \right| \quad \text{for } 1 \le i \le n$$

$$= \left\{ n^3, n^3 - 1, \ldots, 3n^2 - 3n + 1 \right\}$$

$$f^*(u_i v_i) = \left| (i-1)^3 - (n+i)^3 \right| \quad \text{for } 1 \le i \le n$$

$$= \left\{ (n+1)^3, (n+2)^3 - 1, \ldots, (2n)^3 - (n-1)^3 \right\}$$

$$f^*(u_i u_{i+1}) = \left| (i-1)^3 - i^3 \right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{ 1, 7, \ldots, 3n^2 - 9n + 7 \right\}$$

$$f^*(u_n u_1) = (n-1)^3$$

$$f^*(cu_i') = \left| n^3 - (n+2i+2)^3 \right| \quad \text{for } 1 \le i \le n$$

$$= \left\{ (n+4)^3 - n^3, (n+6)^3 - n^3, \ldots, (3n+2)^3 - n^3 \right\}$$

$$f^*(u_i v_i') = \left| (i-1)^3 - (n+2i+3)^3 \right| \quad \text{for } 1 \le i \le n$$

$$= \left\{ (n+5)^3, (n+7)^3 - 1^3, \ldots, (3n+3)^3 - (n-1)^3 \right\}$$

$$f^*(u_i u_{i+1}') = \left| (i-1)^3 - (n+2i+4)^3 \right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{ (n+6)^3, (n+8)^3 - 1^3, \ldots, (3n+2)^3 - (n-2)^3 \right\}$$

$$f^*(u_n u_1') = (n-1)^3 - (n+4)^3$$

$$f^*(u_{i+1} u_i') = \left| i^3 - (n+2i+2)^3 \right| \quad \text{for } 1 \le i \le n-1$$

$$= \left\{ (n+4)^3 - 1^3, (n+6)^3 - 2^3, \ldots, (3n)^3 - (n-1)^3 \right\}$$

$$f^*(u_1 u_n') = (3n+2)^3$$

$$f^*(v_i u_i') = \left| (n+i)^3 - (n+2i+2)^3 \right| \quad \text{for } 1 \le i \le n$$

$$= \left\{ (n+4)^3 - (n+1)^3, (n+6)^3 - (n+2)^3, \ldots, (3n+2)^3 - (2n)^3 \right\}$$

Thus the edge labels are distinct. Hence the graph obtained by duplicating all the rim vertices of Helm

H_n admits a cube difference labeling.

III. Conclusion

It is very exciting to investigate on graph families which are cube difference graphs. Those results are seems easy but it is a big challenge and good experience for us to investigate and prove. Here we have analyzed and proved in details about some results on cube difference graphs such as helm and wheel related graphs such as switching of wheel and helm, duplication of helm and except apex vertex of helm graph are cube difference.

IV. ACKNOWLEDGEMENT

The author is grateful to my guide and thanks to ManonmaniamSundaranar University for providing facilities. I wish to express my deep sense of gratitude to Dr. M. Karuppaiyan my husband for their immense help in studies.

REFERENCES

[1] V. Ajitha, S. Arumugam, and K. A. Germina. “On Square sum graphs” AKCE J. Graphs, Combin, 6, 2006. 1-10.

[2] Frank Harary. “Graph theory” Narosa Publishing House 2001.

[3] J. A. Gallian. “A dynamic survey of graph labeling” The Electronics journal of Combinatories, 17, 2010 # DS6.

[4] J. Shiama. “Square sum labeling for some middle and total graphs” International Journal of Computer Applications (0975-08887), 37(4), January 2012.

[5] J. Shiama. “Square difference labeling for some graphs” International Journal of Computer Applications (0975-08887), 44 (4), April 2012.

[6] J. Shiama. “Some Special types of Square difference graphs” International Journal of Mathematical archives, 3(6), 2012, 2369-2374 ISSN 2229-5046.

[7] J. Shiama. “Cube Difference Labeling Of Some Graphs” International Journal of Engineering Science and Innovative Technology, 2(6), November 2013.

[8] V. Sharon Philomena and K. Thirusangu. “Square and Cube Difference Labeling of Cycle Cactus, Special Tree and a New Key Graphs” Annals of Pure and Applied Mathematics. 8(2). December 2014.