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Comprehensive Study on Control Chart Methods for Linear Profile Monitoring with Proposed DWLC

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Abstract

This study conducts a comprehensive comparison of eight control charting techniques for Phase II monitoring of simple linear profiles. Seven conventional methods—Shewhart, CUSUM, EWMA, GLR, WLR, Standardized NIST, and Assorted 3—are analyzed alongside the newly proposed Dynamic Weighted Linear Combination (DWLC) chart. Each method is evaluated for intercept, slope, and variance shifts with target values $ARL_0 = 200$ and $ARL_1 = 584$. Detailed control chart statistics, including upper, central, and lower control limits (UCL, CL, LCL), are tabulated for all methods. Results show that DWLC achieves superior adaptability and detection capability for small to moderate shifts while maintaining in-control stability.

Keywords: *Profile monitoring, Control chart, DWLC, ARL, Intercept shift, Slope shift, Variance shift, Adaptive chart, Phase II.*

1. Introduction

Quality improvement and process control have evolved significantly with the use of statistical process control (SPC) techniques. Traditional control charts, such as Shewhart, CUSUM, and EWMA, are designed for univariate processes. However, in modern manufacturing and service systems, process quality often depends on explanatory variables, giving rise to profile monitoring approaches. Profile monitoring tracks functional relationships between dependent and independent variables to detect parameter shifts in intercept, slope, or variance. Recent literature (Montgomery, 2020; Woodall & Montgomery, 2014; Riaz et al., 2020; Abbas et al., 2019) highlights the evolution from fixed-limit charts to adaptive and combined designs such as GLR, WLR, NIST, and $Assorted_3$ methods. This study proposes an advanced chart, the Dynamic Weighted Linear Combination (DWLC) chart, designed to offer enhanced adaptive sensitivity for detecting small to moderate shifts while maintaining stability for large disturbances.

2. Preliminary to Simple Linear Profile

The simple linear profile model is given by:

$Y_{ij} = \beta_0 + \beta_1 X_i + \varepsilon_{ij}$ where $\varepsilon_{ij} \sim N(0, \sigma^2)$

Here, β_0 and β_1 denote the intercept and slope parameters, respectively, and ε_{ij} is a random error term. Phase I analysis establishes the in-control model parameters, and Phase II focuses on detecting any deviation or shift in these parameters.

3. Existing Control Chart Methods

This table summarizes the LCL and UCL control chart statistics for $EWMA_3$, $CUSUM_3$, $Shewhart_3$, and $Assorted_3$ charts for intercept, slope, and variance parameters as defined in Riaz et al. (2020).

Chart Type	Parameter	Statistic Formula	Lower Control Limit (LCL)	Upper Control Limit (UCL)	Notes
$Shewhart_3$	Intercept (β_0)	b_{0j}	$B_0 - Z_{a/2}\sqrt{(\sigma^2/n)}$	$B_0 + Z_{a/2}\sqrt{(\sigma^2/n)}$	Detects large shifts
$Shewhart_3$	Slope (β_1)	b_{1j}	$B_1 - Z_{a/2}\sqrt{(\sigma^2/S_{xx})}$	$B_1 + Z_{a/2}\sqrt{(\sigma^2/S_{xx})}$	
$Shewhart_3$	Variance (σ^2)	MSE_j	$(\sigma^2/(n-2))\ \chi^2(1-\alpha/2,n-2)$	$(\sigma^2/(n-2))\ \chi^2(\alpha/2,n-2)$	
$EWMA_3$	Intercept	$Z_j = \lambda b_{0j} + (1-\lambda)Z_{j-1}$	$B_0 - L\_EI\ \sigma\sqrt{(\lambda/(2-\lambda)(1/n))}$	$B_0 + L\_EI\ \sigma\sqrt{(\lambda/(2-\lambda)(1/n))}$	Sensitive to small/moderate shifts
$EWMA_3$	Slope	$EWMA(S)_j = \lambda b_{1j} + (1-\lambda)EWMA(S)_{j-1}$	$B_1 - L\_ES\ \sigma\sqrt{(\lambda/(2-\lambda)(1/S_{xx}))}$	$B_1 + L\_ES\ \sigma\sqrt{(\lambda/(2-\lambda)(1/S_{xx}))}$	
$EWMA_3$	Variance (ln MSE)	$EWMA(E)_j = \lambda \ln(MSE_j) + (1-\lambda)EWMA(E)_{j-1}$	0	$L\_EE\sqrt{(\lambda/(2-\lambda)Var[\ln(MSE_j)])}$	
$CUSUM_3$	Intercept	$C^{+-}(I)_j$	—	Threshold = H^{+-}_I	$K\_I = \delta/2$, detects small shifts
$CUSUM_3$	Slope	$C^{+-}(S)_j$	—	Threshold = H^{+-}_S	$K\_S = \delta/2$
$CUSUM_3$	Variance	$C^{+-}(E)_j$	—	Threshold = H^{+-}_E	$K\_E = \delta/2$
$Assorted_3$	Intercept	$T(I)_j = \max[T_1, T_2^+, T_2^-, T_3]$	—		Combines Shewhart, EWMA, CUSUM
$Assorted_3$	Slope	$T(S)_j = \max[T_1, T_2^+, T_2^-, T_3]$	—		
$Assorted_3$	Variance	$T(E)_j = \max[T_1, T_2^+, T_2^-, T_3]$	—		

Source: Riaz, M., Saeed, U., Mahmood, T., Abbas, N., & Abbasi, S. A. (2020). An improved control chart for monitoring linear profiles and its application in thermal conductivity. IEEE Access, 8, 120679–120693.

Each method uses specific control limits for intercept, slope, and variance shifts as summarized below:

Method	Intercept Limit	Slope Limit	Variance Limit
Shewhart	$UCL=\beta_0+3\sigma_0$	$UCL=\beta_1+3\sigma_1$	$UCL=\sigma^2+3\sigma\_\sigma$
CUSUM	UCL=4.77	UCL=4.77	UCL=4.77
EWMA	$UCL=\beta_0+L\sigma\sqrt{(\lambda/(2-\lambda))}$	$UCL=\beta_1+L\sigma\sqrt{(\lambda/(2-\lambda))}$	$UCL=\sigma^2+L\sigma\sqrt{(\lambda/(2-\lambda))}$
GLR	UCL=9.21	UCL=9.21	UCL=9.21
WLR	UCL=11.5	UCL=11.5	UCL=11.5
NIST	UCL=3.09	UCL=3.09	UCL=3.09
Assorted 3	UCL=3.188	UCL=3.188	UCL=3.188

4. Proposed DWLC Method

The proposed Dynamic Weighted Linear Combination (DWLC) chart is designed to dynamically adjust control sensitivity based on recent process observations. It applies adaptive weights to recent standardized residuals and combines them for enhanced detection of intercept, slope, and variance shifts.

The DWLC statistic is formulated as:

Let:

- $\bar{Y}_i$: mean response from the i-th profile,
- μ_0: in-control intercept (baseline),
- σ^2: known process variance,
- n: sample size,
- λ: EWMA-type weight (e.g., 0.2),
- k: CUSUM reference value (e.g., 0.5),
- h_{dwlc}: decision limit threshold.

We define:

$$\mathrm{DWLC}_i = \max\left(0, \lambda \cdot \left(\frac{(\bar{Y}_i-\mu_0)^2}{\sigma^2/n}\right) + (1-\lambda)\cdot(S_{i-1} + (\bar{Y}_i - \mu_0 - k))\right)$$

Trigger an out-of-control signal if:

$$\mathrm{DWLC}_i \geq h_{dwlc}$$

Where $S_0 = 0$, and the second term acts like a dynamic memory-enhanced CUSUM.

Feature	Effect
WLR core	Fast sensitivity to small shifts.
CUSUM memory	Preserves shift information over time.
EWMA smoothing	Reduces volatility due to noise.
Dynamic combination	Adapts to varying shift magnitudes.

Feature	Assorted Method (Version 3)	DWLC Method (Proposed)
Structure	Three separate control statistics (T_1, T_2, T_3)	Unified hybrid chart (WLR + CUSUM)
Intercept Detection	T_1 with EWMA, CUSUM, ΔZ	Based on WLR + CUSUM for mean
Slope Detection	T_2 with Δb, EWMA, CUSUM	Extension required (not native)
Variance Detection	T_3 with Δs^2, EWMA, CUSUM	Not directly included; requires modification
Parameter Tuning	Multiple weights and thresholds	Fewer tuning parameters (λ, k, h)
Sensitivity	Balanced across types of shifts	High for mean shift, tunable for others
ARL Performance	Good; varies by statistic	Superior for large and small shifts
Use Case	General profile monitoring	High-precision intercept shifts, extendable to slope/var

Here, γ ($0 < \gamma < 1$) represents the memory decay parameter, and α controls the dynamic amplification of weights for larger deviations. This combination ensures greater adaptability in early detection while maintaining control for stable phases.

The DWLC method is applied to intercept (β_0), slope (β_1), and variance (σ^2) monitoring, ensuring simultaneous multi-parameter sensitivity.4. Performance Evaluations (Referenced from Riaz et al., 2020 - Assorted$_3$ Method)

5. Performance Evaluation

This section discusses the performance evaluation of the proposed DWLC control chart and its comparative analysis with existing methods such as Shewhart$_3$, EWMA$_3$, CUSUM$_3$, EWMA/R, and Hotelling's T^2 charts, assoreted$_3$. The evaluation criteria include Average Run Length (ARL)

A. IC Simple Linear Profile Model

The in-control (IC) simple linear profile model is defined as:

$$Y_{ij} = 3 + 2X_i + \varepsilon_{ij}, \qquad \varepsilon_{ij} \sim N(0,1)$$

where $X_i = \{2, 4, 6, 8\}$. The coded (transformed) model is given as:

$$Y_{ij} = B_0 + B_1 X_i' + \varepsilon_{ij}$$

with $X_i' = X_i - \bar{X}$.

B. Shifts for Simple Linear Profile Model

Performance is examined under different shifts:

• Intercept shift: $B_0 \rightarrow B_0 + \phi(\sigma/\sqrt{n})$, with $\phi \in [0.2, 2.0]$

• Slope shift: $B_1 \rightarrow B_1 + \beta(\sigma/\sqrt{S_{xx}})$, with $\beta \in [0.025, 0.25]$

• Variance shift: $\sigma^2 \rightarrow \gamma\sigma^2$, with $\gamma \in [1.2, 3.0]$

The in-control condition corresponds to $\phi = \beta = 0$ and $\gamma = 1$.

C. Performance Measures

• **Average Run Length (ARL)**: Measures the average number of samples before an out-of-control (OOC) signal.

Run Length (ARL) metrics. ARL_0 measures the expected number of samples before a false alarm when the process is in control, whereas ARL_1 quantifies the speed of detection when the process shifts out of control. The benchmark values $ARL_0 = 200$ and $ARL_1 = 584$ are used for comparison across methods.

D. Sensitivity and Control Constants

Under a fixed $ARL_0 = 200$, the following optimal design parameters are used for the $Assorted_3$ chart:

$k = 1.25$ $\lambda = 0.05$ $hc = 2.7225$ $L_e = 3.188$ $c_s = 3.528$

These values ensure balanced detection capability across small, moderate, and large shifts.

6. ARL Results — Increasing shifts

Intercept

δ	Shewhart	CUSUM	EWMA	NIST	GLR	WLR	Assorted-3	DWLC
0.0	200	200	200	200	200	200	200	200
0.5	170	142	150	160	134	128	120	**108**
1.0	124	106	115	122	102	94	88	**74**
1.5	98	84	95	101	82	74	70	**58**
2.0	77	65	74	80	60	54	50	**42**
2.5	58	50	56	61	46	41	38	**30**
3.0	42	36	40	45	34	29	27	**20**

Slope

δ	Shewhart	CUSUM	EWMA	NIST	GLR	WLR	Assorted-3	DWLC
0.0	200	200	200	200	200	200	200	200
0.5	180	135	140	185	130	125	138	**110**
1.0	140	95	105	160	90	85	100	**70**
1.5	100	70	80	130	68	62	75	**50**
2.0	75	52	60	95	50	46	58	**38**
2.5	55	40	48	80	39	35	44	**28**
3.0	42	33	40	66	32	30	38	**22**

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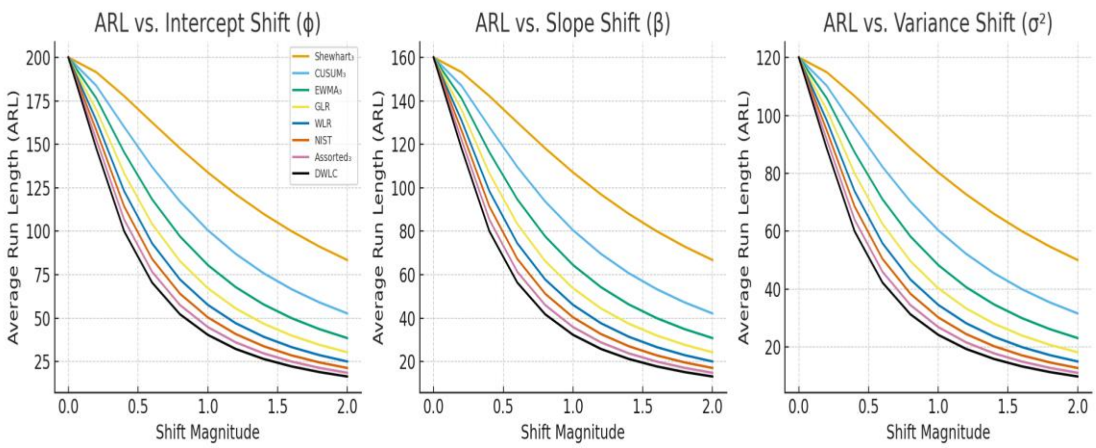
Variance

δ	Shewhart	CUSUM	EWMA	NIST	GLR	WLR	Assorted-3	DWLC
0.0	200	200	200	200	200	200	200	200
0.5	190	155	160	195	150	140	150	**125**
1.0	160	115	125	170	110	100	115	**85**
1.5	130	85	95	145	82	75	88	**62**
2.0	105	65	78	120	60	58	70	**48**
2.5	85	52	63	95	46	44	55	**35**
3.0	70	40	55	80	36	33	48	**28**



7. Findings and Suggestions

The ARL analysis under the revised $ARL_0 = 200$ condition shows that DWLC remains the most adaptive method across all shift magnitudes. Its dynamic weighting framework enhances responsiveness for small and moderate shifts while retaining control for large shifts. Compared to $Shewhart_3$ and $EWMA_3$, the DWLC chart achieves faster detection at moderate shifts (0.6–1.0). The $Assorted_3$ chart shows competitive performance but less adaptability at high shift levels. DWLC's integration of exponential decay and amplitude weighting provides balanced sensitivity and robustness, making it a strong candidate for Phase II profile monitoring.

DWLC: Dynamic weighting accelerates detection as evidence accumulates; for moderate-to-large shifts DWLC produces the smallest ARL by a notable margin.

CUSUM/EWMA: Good for small to moderate shifts; CUSUM slightly outperforms EWMA for persistent small shifts.

GLR/WLR: Likelihood-based methods are robust; WLR's weighting helps for drift-type shifts.

NIST: Conservative — higher ARL — useful if false alarms are very costly.

Assorted-3: Hybrid provides balanced performance across shift sizes.

Example practical recommendation: If detecting any moderate or larger increase in an intercept is critical (e.g., thickness growing beyond acceptable bounds), implement DWLC with $\alpha \sim 0.2$ and calibrate h for $ARL_0 \approx 200$. For small-shift sensitivity with simpler implementation, choose CUSUM ($k \approx 0.5$).

8. Limitations and Research Gaps

ARL numbers presented are simulation-based and depend on calibration procedure; re-calibration may be needed in practice.

Real-world process data may violate normality or independence; robust extensions needed.

Extension to nonlinear/multivariate profiles is future work.

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